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Agua Para La Vida

# Air In Water Pipes 

# Second Edition, (10-2003) <br> A Manual for Designers of Spring-Supplied <br> Gravity-Driven <br> Drinking Water Rural Delivery Systems 

by

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AGUA PARA LA VIDA is a non-profit organization with branches in USA and in France and its purpose is to help remote rural communities acquire enough safe drinking water. Our members are active in Nicaragua where we have designed and helped build drinking water supply systems since 1987. We expect to participate in answering this primary need in other countries as well.

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GILLES CORCOS is one of the founders of Agua Para La Vida. He has been a Professor of Engineering at the University of California, Berkeley, specializing in the mechanics of liquids and gases for thirty two years. His first drinking water project dates back to 1962 . The material for this manual was gathered in the field in Nicaragua and in the Laboratory in Berkeley. Dan Mote, then Chairman of the Mechanical Engineering Department supported the work. Several classes of Engineering Seniors chose this very topic for their senior laboratory class and contributed useful data. The author is grateful to them all.

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## INTRODUCTION

When you design a pipeline for a gravity flow water system, you usually assume that the water flow will fill the pipe. In this case, the flow rate out of the pipe is controlled by the available head, the length, diameter and roughness of the pipe and the so-called minor losses due to various obstructions (contractions, elbows, expansions, tees and especially valves). We will call this the full-pipe or friction controlled case. It is adequately explained in many textbooks and manuals* . But if for some reason the pipe is not completely full of water, the relation between head available and flow rate is very different. This will happen in several cases. For instances:

- When you first turn on the water in a new installation with dry or partially filled pipes.
- If you have cavitations somewhere in your circuits (too much suction).
- If the pipe is fed by a spring through a small spring box and the output of the spring is smaller than the one for which you designed the pipe system.

Now the general belief is that if you have air in the pipes you need to get rid of it so that the pipes will run full. This is because the presence of air often increases the head required for a given flow. In fact, it is not at all rare that this air acts as a block so that no water at all comes out at the end of the pipe.

On the other hand, it turns out that very frequently, in systems with springs of small, uncertain or variable output, there is a big advantage in operating with air in the pipes. The advantage is that you can design such a system so that it will operate like a canal, instead of a pipeline: within limits, which you can easily calculate, it will deliver to the end of the line whatever flow rate is provided by the spring and this without having to adjust a valve- without controls.

This manual is written to help you deal with air in water pipes. In particular it will make it possible for you to :

- Understand the problem of starting with dry pipes.
- Predict what will happen if your water supply is or becomes smaller than you assumed in your calculations ( for a friction-controlled system).

[^0]- Design deliberately smooth operating systems in which air is almost always present in the pipes.

Throughout this manual, we assume that even though the spring output will vary in time one of your primary objectives is to convey to the distribution tank through the conduction line, a flow rate that is always the full spring output up to a maximum of your choice.

The manual addresses itself at the same time to two kinds of designers:

- Those who have not had the unpleasant experience of problems with air making their conduits misbehave.
- Those who are leery of air problems and dutifully place an automatic air valve at every intermediate high point of their conduits.

First, the hydraulic background necessary to understand this subject is presented. Then, the method for predicting and designing with air in the pipes is given. Finally, the manual presents a number of examples that will help you use this material and get on top of the subject.

You will find among these examples some friction-controlled designs that get in trouble when the flow rate of the spring is only a little bit less than the design value, as well as cases in which there is no air trouble, no matter how much the flow supplied to the pipe by the spring is reduced. After you have followed these examples, you will be able to predict whether your friction-controlled design will give you trouble in a specific case. You will also be able to modify your designs so as to eliminate problems with air in the pipes.

The examples make it clear that the problem is not to choose between a frictioncontrolled full pipe design and a mixed air-water design but rather to adapt the design to the probability that air will be present in the pipes some of the time.

The standard way to deal with the possibility of blockage with air pockets is to systematically place an automatic valve at every high point of the conduction line. We don't favor this solution for the following reasons:

1) As will be explained in the text, when such a valve operates in a "supercritical" part of the circuit with transient water mixed with air, it constantly and abruptly opens and closes. That limits its life.
2) Some circuits require no valves at all and no special provisions to deal with ingested air. Thus, for these cases, why increase complexity and costs?
3) The placement of air valves near high points is a matter of some delicacy: they have to be placed near the start but within the air pockets, which develop (if there is any flow) only downstream of the high points and the location of the high point is rarely obvious. This is an additional argument to avoid placing blindly unnecessary valves.

The material which follows is arranged so that the general foundations are given in Chapter 1. The way to proceed in a design is given in Chapter 2. This chapter is the one that tells you how to deal with air. In other words, Chapter 2 is the "how to" part of the manual while Chapter 1 is a reference of "why" chapter. Any supplementary information required for the design is found in Appendix A, whether it is new information or available in other books or manual. Chapter 3 presents the examples that illustrate the material. Appendix B makes a few more specialized points which would perhaps be confusing in the main text. But these can be read later.

The solutions suggested in Chapters II \& III are, of course, not the only ones, perhaps not even the best ones. After you have examined this material, no doubt you will choose your own. The important thing is to have in hand enough elements to make an enlightened choice.
Note: It is possible (though often tedious) for you to carry out a suitable design without using the equations that appear in the text, for examples, by only adding, subtracting, multiplying, dividing and using the tables. The only exceptions are the two formulas of Appendix A-III. This is shown in Chapters II \& III. But you can also download from the web, an APLV program called Air-in Pipes that will lead you rapidly (and somewhat more accurately) through your conduction pipe design. (See our Web site, www.aplv.org or request the program from aplv@igc.org. This Visual Basic Excel program is due to Charlie Huizenga with suggestions from Katherine Force, Mathieu Le Corre, Jim Stacey and Gilles Corcos.

The second edition of this manual is the outcome of twelve years of continuous use of the first. It incorporates some changes in the presentation and adds attention to some cases that have occurred in the field more often than what we previously anticipated.

## SYMBOLS

$\mathrm{h}=$ head (meters)
$\mathrm{H}=$ height (meters); $\mathrm{H}_{\mathrm{AB}}=$ height difference between points A and B (meaning $\left.H_{A}-H_{B}\right) ; Q=$ flow rate ( $\mathrm{m}^{3} / \mathrm{s}$ unless otherwise specified). Subscripts $1 \& 2$ refer to two points along the pipe with 1 upstream of 2 . Letter $S$ refers to the spring or spring tank. Letter T refers to the pipe outlet or distribution tank. Other points along the pipe are indicated in the sketches.
$\mathrm{h}_{\mathrm{a}}=$ head available $=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}$
$h_{t}=$ trickle height defined in the text. Appendix A shows you how to calculate it.
$\mathrm{h}_{\mathrm{f}}=$ friction head loss. You can use table A1 or the formulas in Appendix A to calculate it.
$\mathrm{h}_{\mathrm{f} 1}=$ friction head loss for $\mathrm{Q}=\mathrm{Q}_{\mathrm{c}}$
$h_{\mathrm{r}}=$ maximum head required $\mathrm{h}_{\mathrm{r}}=\mathrm{h}_{\mathrm{f}}+\mathrm{h}_{\mathrm{t}}$ if Q is smaller than $\mathrm{Q}_{\mathrm{C}}$ and $\mathrm{h}_{\mathrm{r}}=\mathrm{h}_{\mathrm{f}}$ if Q is greater than $\mathrm{Q}_{\mathrm{C}}$
$\mathrm{Q}=$ flow rate $\left(\mathrm{m}^{3} / \mathrm{s}\right)$
$\mathrm{Q}_{\mathrm{c}}=$ The lowest critical flow rate: The flow rate at which in a horizontal part of the pipe a long bubble is carried downstream.Its value depends almost only on pipe diameter(see equation 2 and table A2).
$Q_{s}=$ The highest critical flow rate: The flow rate at which a long bubble will be carried downstream, regardless of the slope of the pipe
$\mathrm{Q}_{\max }=$ maximum expected output (flow rate) of the spring. $\mathrm{Q}_{\text {min. }}=$ minimum expected output of the spring.
$\mathrm{Q}^{*}=\mathrm{Q} / \mathrm{Q}_{\mathrm{c}}$ or $\mathrm{Q} / \mathrm{Q}_{s}$. If $\mathrm{Q}<\mathrm{Q}_{\mathrm{c}}$ the flow is called subcritical. If $\mathrm{Q}>\mathrm{Q}_{\mathrm{s}}$ it is called supercritical. When $\mathrm{Q}_{\mathrm{c}}<\mathrm{Q}<\mathrm{Q}_{s}$ the flow is transitional.
$\mathrm{L}=$ length of a pipe line. $\mathrm{L}_{\mathrm{ST}}$ length along the pipeline from the spring to the distribution tank. $\mathrm{L}_{\mathrm{AB}}=$ length of pipe between points A and B ... etc..
$\mathrm{V}=$ water (section-averaged or discharge) velocity; $\mathrm{m} / \mathrm{s}$.
$\mathrm{g}=$ acceleration of gravity $\left(9.81 \mathrm{~m} / \mathrm{sec}^{2}\right)$.
$A=$ cross sectional area of the pipe interior.

## CHAPTER I

## AIR IN THE PIPELINES

Air may be found in water pipelines mainly as large, stationary pockets, or as large or small moving bubbles.

When air collects in parts of the pipeline, without moving, the water may be blocked by the air so that no water flows or it may circulate past the pockets of air by flowing underneath these pockets. You will learn to figure out which will happen in a particular case.

When water flows sufficiently fast, air pockets are not able to remain still and they will be chased down the pipe along with the water. Then, the presence of air in the pipe will not affect the delivery of the water. You will also be able to determine when that happens in any particular case.

Stationary Air Pockets. You may first run into this problem when you fill for the

first time a newly constructed gravity flow pipeline since this pipeline starts out at least partially full of air. If, as in figure I-1, the profile has a local maximum (point B) between spring $S$ and tank $T$, as you allow a small flow of water out of $S$, the water will accumulate at the low point $A$ and then, fill the pipe on both sides of $A$
(Figure I-1a). Progressively, air is chased out of this section of the pipe until there is no more air between A and B and the water reaches the level of the bottom of the pipe at B (Figure I-1b). The section BC' is still full of air and the water will now trickle down towards $C^{\prime}$ below the air. This air will not be flushed out by a small water flow rate. We will call the stationary air pocket above the trickle of falling water an air sock. The trickle of water below the air sock soon fills the bottom of the pipe at $\mathrm{C}^{\prime}$ so that the air between B and $\mathrm{C}^{\prime}$ is now trapped and isolated from the atmosphere: the sock is closed (Figure I-1b).

Now, the pressure throughout the sock downstream of B has to be uniform (because hydrostatic pressure variations are negligible in a gas), and this forces a uniform pressure in the thin stream of water flowing below the air sock. This is the origin of the head loss due to the presence of the sock: Between B and the end of the sock, the water loses potential energy (height) and there is no corresponding increase in pressure head since the pressure remains the same in the stream below the air sock and the kinetic energy (velocity head) is the same at the beginning and the end of the sock.

The head loss caused by the presence of the sock is the difference between the elevations of the beginning and of the end of the sock.

If there are several local high points as in Figure.I-2, as you continue to fill the pipe, more air socks appear downstream of these high points so that more head is lost. The total head loss due to all the socks is the sum of the individual sock head losses.


Figure I-2
In Figures I-1c and I-2, note that while the top of the air socks remains at the level of the local high points, such as B or D, the bottom does not have to remain at the local low points because as you keep filling the pipe, the hydrostatic pressure in the socks increases. This compresses the air in the socks which causes the volume of
the socks to decrease. As a result, the socks become shorter and the level of the sock bottom rises from $\mathrm{C}^{\prime}$ to C and from $\mathrm{E}^{\prime}$ to E .

## As You Fill The Pipe Will the Water Flow out At All?

If you keep releasing water slowly from the spring, the water level in the pipe below the spring may reach the spring tank level S and spill there before it comes out at the end of the pipe at level T. This is the case shown on Figure I-1c. In that case no water can be delivered to the downstream tank until some air has been purged out of the socks. This will happen if the available head, $h_{a}=H_{S}-H_{T}$, is smaller than the sum of the heights of the socks.

Or the water may come out at $T$ before it has backed up to the level of $S$. In this case you probably still want to purge the air out of the socks but even before you do so, some water will flow out the end of the pipe. This will happen when $h_{a}=$ $\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}$ is larger than the sum of the heights of the socks.

This brings out the importance of figuring out the sum of the heights of the socks. The way to do that is given later in Chapter II.

So far, we have imagined that only a very small quantity of water is released at the spring. Now, we imagine that the spring output is larger, though always limited. In other words, we don't have a large reservoir or lake at the spring site. We can see that the flow of water and the discharge at T can be limited in one of the following two ways:

It can be limited by the head loss in the pipe. This head loss increases with the flow rate so that there is a flow rate for which the head loss is equal to the head available and clearly no matter how much water comes out of the spring the amount that flows through the pipe cannot exceed the flow rate for which the head available is equal to the head loss. The rest of the spring output merely overflows at the spring tank.

Or the discharge may be limited by the output of the spring. This happens whenever this output is smaller than the flow rate for which the head losses equal to the head available.

In the first case (flow rate limited by the head losses), the standpipe below S is normally full of water.

In the second case (flow rate limited by the spring output), the flow out of the spring tank does not fill the pipe and we start below $S$ with what is roughly speaking a waterfall. This is important because in general, air bubbles are created by the waterfall and they are carried (to a greater or lesser extent) by the water down the pipe so that in this second case air is supplied to the pipe along with the
water. Sometimes this does not matter but as we will soon see, there are frequent cases for which this new source of air causes problems.

## The Critical Flow

We now return to the air socks which we assume we have not purged out. It turns out that there is a special flow rate we call critical flow rate, $Q_{C}$, which is fixed by the pipe diameter in the region of the socks.

The system may only be capable of a flow rate less than the critical flow rate $\mathrm{Q}_{\mathrm{c}}$ either because the flow out of the spring is less or because the combined losses due to friction at $\mathrm{Q}=\mathrm{Q}_{\mathrm{C}}$ and the air in the socks exceed the available head.

Or the system is capable of a flow rate larger than $Q_{c}$ because the flow out of the spring is larger and the head available is larger than the sum of friction head required for $\mathrm{Q}=\mathrm{Q}_{\mathrm{C}}$ and the head loss from air in the pipe.

Now the critical flow rate $\mathrm{Q}_{\mathrm{c}}$ has the following physical meaning:
If the flow rate $Q$ of which the system is capable is smaller than $Q_{C}$, air socks will remain in fixed locations downstream of the high points. Their tops will remain level with the high points. Their bottoms will have a level which depends on the amount of air that has found its way to the sock and on the pressure within the sock. The loss of head they will cause is still the sum of the heights of the socks.

If the flow rate Q of which the system is capable is greater than $\mathrm{Q}_{\mathrm{C}}$, the air socks will be chased out of the horizontal part of the pipe and any additional air coming from upstream will also circulate through that zone without stopping there. The critical flow rate only depends on the diameter of the pipe in the region of the sock: It is reached at a high point where

$$
\begin{equation*}
Q_{c}=0.38 d^{5 / 2} g^{1 / 2} * \tag{1a}
\end{equation*}
$$

But stationary socks will not be safely expelled downstream past the following sloping part of the pipe until the flow rate Q exceeds

$$
\begin{equation*}
\mathbf{Q}_{\mathrm{s}}=\mathbf{0 . 5 0 d ^ { 5 / 2 } d ^ { 1 / 2 } *} \tag{1b}
\end{equation*}
$$

Where $d$ is the inner diameter of the pipe at the location of the sock and $g$ is the acceleration of gravity. If $Q$ is in cubic meters per second, equations (1a) and (1b) can be rewritten

$$
\begin{equation*}
Q_{c}=1.19 d^{5 / 2} \tag{2a}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{s}=1.57 d^{5 / 2} \tag{2b}
\end{equation*}
$$

$\mathrm{Q}_{\mathrm{c}}$ and $\mathrm{Q}_{\mathrm{s}}$ are given in table (A2, Appendix A ) for a few pipe diameters. Flow rates when larger than $\underline{Q}_{\underline{S}}$ are called supercritical, and when lower than $\underline{Q}_{\underline{\mathrm{C}},} \underline{\text { subcritical. }}$

## Can We Evaluate the Additional Head Loss Due to the Socks?

In full pipes with no air socks, a familiar energy equation allows us to determine the flow. It is:

$$
8 \mathrm{Q}^{2} / \mathrm{d}_{1} 4 \mathrm{~g}+\mathrm{h}_{1}+\mathrm{H}_{1}=8 \mathrm{Q}^{2} / \mathrm{d}_{2} 4 \mathrm{~g}+\mathrm{h}_{2}+\mathrm{H}_{2}+\mathrm{h}_{\mathrm{f}}
$$

The first term on the left is the "kinetic energy" (velocity energy). The second is the pressure energy. The third is the "potential energy" (energy due to height). The subscript 1 refers to a section upstream and the subscript 2 to a section downstream. The equation says that the sum of the three energy terms at the downstream section is less than the sum of the three energy terms at the upstream section because there is a loss $h_{f}$ due to friction between 1 and 2 . This loss is always positive. You can calculate $\mathrm{h}_{\mathrm{f}}$ if you know the values of $\mathrm{Q}, \mathrm{d}$, and the distance between 1 and 2 (see Appendix A). Now, as we have just seen, when there are air socks between 1 and 2 the energy equation must include an additional loss term on the right hand side of the equation. This term can be calculated only when the difference between the elevations of the beginning and the end of all socks are known. However, this is the case only as you fill dry pipes and not later because air can leave the air pockets as small bubbles and also come in from upstream to replenish them. Therefore, in general, you won't know how much air there is in the pipe. What is known is that the initial head loss due to the socks, which is found when you first fill the dry pipes (which we will later call the trickle height, $\mathrm{h}_{\mathrm{t}}$ ), is the maximum air sock head loss you will encounter. You will usually design for this worse case situation.

You should keep in mind that: The head loss due to an air sock containing a given mass of air will not change drastically, whether the flow rate is very small or not, (it will slowly increase with flow rate) as long as the flow rate is less than the critical flow rate, $Q_{\mathbf{c}}$. For $Q$ larger than $Q_{S}$, the trickle height head loss disappears because the air is chased out.

## The sock head loss is not just a starting problem

Whether the flow is subcritical or supercritical, air bubbles and air pockets can be carried from the area of the pipe below the spring towards the location of the air socks as long as the head required is less than the head available. If the flow is subcritical this means that socks can be replenished with air after they have been flushed out manually.

## Moving Air Pockets And Bubbles.

What happens when Q exceeds $\mathrm{Q}_{\mathrm{S}}$ is that since the water flow is now large enough to flush out the air sock, the head required suddenly decreases ( $h_{t}$ disappears). Now, if the head available is large enough to cause Q to exceed $\mathrm{Q}_{\mathrm{S}}$ before the sock is chased out, an even greater flow rate will occur afterwards. If the output of the spring is sufficient to keep up with this greater flow rate, i.e. the flow rate determined by the head available with ordinary friction losses, the pipe will fill up with water and remain full. But if not, i.e. if the output of the spring falls short of the flow rate which the pipe can sustain with the available head $h_{a}$ when the pipe is full, air will almost always be entrained by the water at the beginning section of the pipe and will travel down the pipe. Some details of this entrainment will be given presently. Keep in mind that no matter how complicated the situation appears, as long as $\underline{Q}$ remains larger than $Q_{\underline{S}}$, the head requirement becomes the one you would calculate for the same Q if the pipe were full.

## Entrainment of air in subcritical and supercritical flows when the spring output limits the flow rate.

Assume now that the flow rate out of the spring has decreased so that it requires less head (for a given pipeline) than the head available. One might think that in this case, the level of the water in the pipe below the spring box would simply adjust itself so that the head of water just balances the head required for available flow rate. In fact this happens only when the flow rate is extremely small (compared to $\mathrm{Q}_{\mathrm{c}}$ ). Instead, if the pipe is not full below the spring, as Q increases, the current is increasingly able to carry air bubbles (created by the fall of water from the spring) downstream with the water. For flow rates much smaller than $Q_{C}$, only very small bubbles are carried downstream. For Q approaching $\mathrm{Q}_{\mathrm{C}}$, bubbles merge and larger bubbles are carried downstream. These can create or add to existing air socks. This normally takes a long time- up to many days. If Q is larger than $\mathrm{Q}_{\mathrm{S}}$, while still smaller than the Q requiring the full head available, air will still be brought in from below the spring but will not remain in any fixed, stationary sock. Instead, it will be flushed out either periodically (when Q is only a little larger than $\mathrm{Q}_{\mathrm{c}}$ ) or more steadily (for greater flow rates). Now the flow down the first section of pipe is very complex-looking, full of pools and cascades. Air bubbles or pockets of various sizes now circulate through the pipe with the water. This has no adverse effect on the operation of the pipeline as long as the spring is able to supply a flow rate larger than $\mathbf{Q}_{\underline{\mathbf{s}}}$. However, if the spring output decreases below $\mathrm{Q}_{\mathrm{c}}$, the air in transit through the pipe will accumulate again in the sock areas downstream of the local highs and contributes a new head loss. So:
The most frequent way to get in trouble with air socks after the system has been started is for the flow to decrease from supercritical to subcritical as a result of a decrease in the output of the spring. In this case your flow may be completely shut out.

## Another form of trouble is subcritical flow which has been started by bleeding the air socks once while the spring output is small. As a result, air slowly returns to the sock area. The capacity of the conduit does not change and the level of the water rises in the pipe towards the spring. If the spring flow rises the air in the socks, it will prevent the pipe from delivering it downstream.

The head requirement (if you start with empty pipes) is made clear in a graph, such as figure I-3.


This graph is appropriate even when there are several local maximae in the pipe profile as in Figure I-2, but only if the pipe diameters downstream of the several maximae are all the same. It shows the maximum head required on the vertical axis and the flow rate on the horizontal axis. When Q is less than $\mathrm{Q}_{\mathrm{C}}$, this head required is the sum of two terms: The trickle height $h_{t}$, and the friction head loss $h_{f} . h_{f}$ is the curve which starts as a dotted line and which rises steadily. It increases with flow rate. When $Q$ is larger than $Q_{c}, h_{r}$ is equal to $h_{f}$ because $h_{t}$ has disappeared (no sock losses).
When there are several socks and downstream of the socks, the pipe diameters are not the same, the sudden decrease in $h_{t}$ as Q increases occurs in steps as in Figure I-4. The first step occurs at the value of $Q$ equal to the value of $Q_{C}$ for the smallest pipe diameter: The sock with the smallest diameter looses its air first.


Figure l-4
Now we return to the case of a single pipe diameter as in Figure I-3.
Let us now indicate the head available, $\mathrm{h}_{\mathrm{a}}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}$ as a horizontal line on the same graph (Figures I-5ab)


Figure l-5a
The smallest value of Q for which this line crosses the head required curve is the maximum flow rate, which the pipeline can deliver. If as in Figure I-5a, the horizontal line $h_{a}$ lies above the curve $\mathrm{h}_{\mathrm{r}}$ for low values of Q , and crosses it only once, the situation is simple: the pipeline will accommodate the flow rate of the spring from $\mathrm{Q}=0$ to $\mathrm{Q}_{1}$, the crossing point, and if the spring output is larger than $\mathrm{Q}_{1}$ the excess will spill at the spring.


Figure I-5b
If as in Figure I-5b, the horizontal line $h_{a}$ is lower than $h_{r}$ even at $Q=0$, water will not flow at all unless enough air is first bled out of the socks so that the curve $\mathbf{h}_{\mathbf{r}}$ falls below $\mathbf{h}_{\mathbf{a}}$. If you do bleed the air, you will be able to have a supercritical flow up to $\mathrm{Q}_{2}$, provided of course that the spring output is sufficient.

These are only two possible situations used to illustrate the meaning of these curves. In Chapter II, you will learn that there are several other cases and that each normally leads to a different design. It is helpful to classify these cases. They depend mostly on the relative magnitude of the head available $h_{a}$, of the trickle height $h_{t}$ and of the friction head loss when $Q=Q_{s}$ which we call $h_{f 1}$.

Here we run into a difficulty which may have already started to confuse you: Both $\mathrm{h}_{\mathrm{f}}$ and $\mathrm{Q}_{\mathrm{S}}$ depend on pipe diameter which cannot be chosen before we classify our case. So how are we going to proceed? Here it goes!

## The ability for a conduit to run supercritically is nearly only a matter of average slope.

We have seen that when the flow is subcritical in the sock regions it cannot get rid of the air in the socks whereas when it is supercritical it will. It follows that only for subcritical flows, shall we have to relieve the pockets of their air. Now, for a flow to run supercritically, it must have at least enough head to overcome friction at the critical flow rate. We presently make two simplifications that allow us to translate approximately that condition into a very simple one.

The first simplification is to assume that the conduit has a single diameter over its whole length, ignoring for now the control that we gain by choosing a different diameter in the socks regions and in the regions where the pipe runs full. That option we will use later when we want to slip from one category to another.

The second simplification is an approximation: the head loss by friction per unit length is often expressed as:

$$
\begin{equation*}
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.0826 \mathrm{Q}^{2} \mathrm{f} / \mathrm{d}^{5}, \tag{3}
\end{equation*}
$$

where $Q$ is flow rate in $\mathrm{m}^{3} / \mathrm{sec}$., d is the inner pipe diameter in meters; $\mathrm{h}_{\mathrm{f}} / \mathrm{L}$ is the head loss per unit length (non-dimensional) and (3) defines f as a non dimensional friction coefficient whose value depends on Reynolds number and pipe roughness.

For PVC pipes the roughness is very small and f varies relatively little over a wide range of diameters, and water flow rates. For smooth pipes the approximation consists in using an average value for f that we take to be 0.026 . Under these conditions since

$$
\mathrm{Q}_{\mathrm{s} .}=1.57 \mathrm{~d}^{5 / 2}
$$

we have simply:

$$
\begin{equation*}
\mathrm{h}_{\mathrm{f} 1} / \mathrm{L} \cong 0.0053 \tag{4a}
\end{equation*}
$$

where $h_{f 1} / L$ is the minmum head loss incurred per unit length for a supercritical flow. In other words, granted our assumptions, if the average pipe slope exceeds something like $1 / 2$ of $1 \%$ and the pipe diameter is chosen (as it must) for pipe friction per unit length to balance average pipe slope for maximum flow, the flow can run supercritically provided it can reach that flow rate (meaning: provided it is not prevented by trickle height losses from reaching critical velocity). On the other hand, with

$$
\begin{align*}
\mathrm{Q}_{\mathrm{c}} & =1.19 \mathrm{~d}^{5 / 2} \\
\mathrm{~h}_{\mathrm{f} 2} / \mathrm{L} & =0.0030 \tag{4b}
\end{align*}
$$

where $h_{\mathrm{f} 2} / \mathrm{L}$ is the upper bound of a head loss per unit length which guarantees a subcritical flow. Thus, for slopes smaller than 0.003 there is no way under the assumptions for the conduction line to run supercritically.

Note that if the flow rate is smaller than that for which friction loss/unit length equals average slope, a slope > . 0.0053 does not guarantee that a supercritical flow can be maintained.

We shall classify a conduction line for which $h_{a} / L>0.0053$ as a case A or a potentially supercritical case and a conduction line for which $h_{a} / L<0.0030$ as a case B or subcritical conduction line. In the computer program that carries out the steps outlined in this manual a more accurate friction calculation is employed to classify the conduction line.

## Remarks

1) Since the friction factor is not really fixed as assumed but varies in practice for critical flow by perhaps as much as + or $-20 \%$, the boundaries of the band between supercritical and subcritical slopes will vary in the same proportions.
2) The notion of a subcritical flow rate has physical meaning only when the water runs over air pockets. Therefore, by choosing pipe diameters judiciously (i.e. large enough) in the sock region and adjusting them elsewhere for the desired friction head loss, you can always make sure that the flow is subcritical in the sock regions if you so desire. This point is taken up in Chapter 2.
3) In the approximate method above for determining the friction head loss at critical flow as well as in the more accurate calculation carried out by the software program, the length of pipe used to get $\mathrm{h}_{\mathrm{f} 2}$ from $\mathrm{h}_{\mathrm{f} 2} / \mathrm{L}$ is that part of the pipe upstream of the relevant sock which is running full, the part occupied by previous socks contributing to $h_{t}$ but not to $h_{f 2}$.
4) If

$$
0.003<h_{a} / L<0.0053
$$

you can test the suitability of operating supercritically by selecting smaller diameters downstream of high points (see discussion of case A2, Chapter II). But in practice, these marginal cases are best handled subcritically.

## Chapter II

## A) Required design input:

- The maximum desired conduction line flow rate $\mathrm{Q}_{\max }$ and the minimum spring output observed $\mathrm{Q}_{\text {min }}$, if available.
- $\mathrm{H}_{\mathrm{S}}$ and $\mathrm{H}_{\mathrm{T}}$ as well as the relative elevation and distance from the spring along the ground of all local low points and high points, (A,B, C', D, E' ....etc, T ). In addition, enough points along the proposed conduction line to be able to define with satisfactory accuracy the height of any point as a function of its distance along the line.
- When the survey is carried out, it is useful to mark the high points with a stake. It also proves useful to survey a few points near the high point, so as to determine how far from it on either side the ground slope reaches, say 1 degree. This will avoid misjudging the location of the high points which can be a serious mistake. Horizontal coordinates (North-South and East West) are not directly used in the design but are nevertheless frequently useful also.


## B) You have a choice:

You may proceed as indicated below or you may use the APLV Air in Pipes (version 4.0e) software program which will perform all these steps for you. However we advise you to examine carefully the content of this manual failing which you won't understand the logic of the solutions proposed. A test of your understanding is to carry out a few of the simpler designs by hand and to compare them with the software program answers.

## C) Design:

This is the design that allows the system to deliver automatically to the tank a variable spring flow rate. It avoids trouble with air in the course of the system operation. Such a design in one of its several versions is always possible if the spring tank is higher than any other point of the pipeline. For the same topography and flow rates, more than one solution may be possible. In that case cost and for instance, the availability of automatic air-purging valves should inform the choice.

## Can you avoid the Problem?

A) If there is no intermediate high point there will be no air blockage and whatever the flow rate out of the spring, you may proceed as if the pipe were always full.

In this case, you should use $\mathrm{Q}_{\text {max }}$ as Q , $\mathrm{L}_{\text {ST }}$ as L and $\mathrm{h}_{\mathrm{a}}$ as $\mathrm{h}_{\mathrm{f}}$ in the friction tables A1.
But mind the execution! Watch out for flat areas where laying the pipes carelessly might result in a slight high point which could yield entirely different results (see example 5b, chapter III).

## B) There are intermediate high points.

If the vented point $\mathbf{T}$ is lower than a local high upstream, you can eliminate the sock downstream of this high if you wish by venting the pipe there with a small break-pressure tank. This has the effect of moving the point $S$ to that high point. However, this solution may turn out to be more expensive than alternative ones so do not stop there. Note also that a break-pressure tank or a vented T are not equivalent to an automatic air valve: The first two impose atmospheric pressure at all times whereas in the air valve, water communicates with the atmosphere only when it is located at the head of an air sock.

If not: the next step are these:
a) Calculate the head available, $h_{a}$, from

$$
\mathrm{h}_{\mathrm{a}}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}
$$

b) Calculate the trickle height, $\mathbf{h}_{\mathrm{t}}$ (for supercritical flows):

The trickle height $h_{t}$ is the largest head loss due to the greatest possible amount of air trapped in a pipe line when the flow rate just reaches the critical flow rate $Q_{C}$. In general, this is not precisely (somewhat larger than) the head you have to overcome to start the water flowing with the pipe full of air because when the flow rate is $\mathrm{Q}_{\mathrm{c}}$, the friction in the full parts of the pipe line tends to decrease the pressure in the socks so that they expand somewhat. But it is the head loss of importance for our designs. The procedure calculates the vertical extent of each sock and then adds them up.


Figure II-1
Draw the pipeline to scale. The only convenient way is to use length along the pipe for your horizontal scale and to exaggerate your vertical scale. Choose a level datum and record the heights of all points such as S,T, A.B,C' etc...with respect to the datum. Record the distance along the pipeline of all these points. Call $\mathrm{L}_{\mathrm{SB}}$ the length of the section between $S$ and $B$, etc...

In determining the trickle height, assume:

- that the initial volume of air is that contained in sections such as $\mathrm{BC}^{\prime}, \mathrm{DE}^{\prime}$, etc...(as when you start with dry pipe). This a conservative assumption but the only absolutely safe one.
- that all the available head may be used to calculate the pressure within the socks ( this means that the reach from $S$ to $B$ is full).

Step 1. Record the quantities: $\left(\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{C}^{\prime}}\right) ;\left(\mathrm{H}_{\mathrm{D}}-\mathrm{H}_{\mathrm{E}^{\prime}}\right)$...etc. and the length that go with these points: $1_{1}{ }^{\prime}=\mathrm{L}_{\mathrm{BC}}{ }^{\prime} ; 1_{2}{ }^{\prime}=\mathrm{L}_{\mathrm{DE}^{\prime}} .$. etc.

Step 2. Calculate the maximum head $h_{1}$ in the first air sock. This is often almost

$$
\mathrm{h}_{1}=\left(\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}\right)-0.0053 \mathrm{~L}_{\mathrm{SB}} .
$$

Note that this formula assumes that the friction is experienced at the critical flow rate. If the flow is supercritical that is the relevant friction for the pipe flow to overcome but if the flow is subcritical, this assumption will underestimate the pressure $h_{1}$. We will see later that this inaccuracy has no consequence.

Step 3. Calculate the change in volume of the air sock due to the compression by the head $h_{1}$. The calculation assumes that the temperature of the air is the same before and (eventually) after the compression (See Appendix B 1). The ratio of the volume $\mathrm{v}_{1}$ after compression to $\mathrm{v}_{1}$ ' before compression is

$$
\mathrm{v}_{1} / \mathrm{v}_{1}^{\prime}=10.4 /\left(10.4+\mathrm{h}_{1}\right)
$$

where $h_{1}$ is expressed in meters.
We will assume that the pipe diameter does not change between B and $C^{\prime}$ So the length of the sock is reduced in the same ratio:

$$
\mathrm{l}_{1}=10.4 \mathrm{l}_{1}{ }^{\prime} /\left(10.4+\mathrm{h}_{1}\right)
$$

Knowing $1_{1}$, you can lay out the segment BC along the pipe (along the horizontal in your scale drawing) and find the height of the point C on the profile.

Step 4. The head at $C$ is the same as the head at $B=h_{1}$. The head in the next pocket (between D and E ) is:

$$
\mathrm{h}_{2}=\mathrm{h}_{1}+\left(\mathrm{H}_{\mathrm{C}}-\mathrm{H}_{\mathrm{D}}\right)-.0055 \mathrm{~L}_{\mathrm{CD}}
$$

If $h_{2}$ is a positive number proceed to step 5 .

## If $h_{\mathbf{2}}$ is negative: Step 4a.

It may happen that $h_{2}$, (and/or $h_{3}, h_{4}$ etc...) is a negative number, (a pressure less than atmospheric). In this case, you should stop your calculation of the trickle height: you have to modify your system because it could cavitate, which means generate water vapor at these low-pressure points which would result in a failure of your gravity delivery. In any case, we systematically avoid negative pressures. The remedy is:

- either to place an automatic air valve "at" (see appendix AIII for proper automatic valve location) one or more of the previous high points;
- or if the high point in question is higher than the tank, to provide break pressure tanks there.

In the first case (automatic air valves $=\mathrm{AAV}$ ), you will recalculate the pressures by assuming that there are no socks between the AAV and the following low points (e.g. if $\mathrm{h}_{2}$ were negative before placing the AAV at B , afterwards, the pressure at D would be simply $\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{D}}$ ).

In the second case (break pressure tanks = BPT), you will start from each BPT as though it were the source. For instance, if pressure $h_{2}$ was negative before placing a BPT at B , afterwards, $\mathrm{h}_{2}$ would be $\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{D}}$.

Step 5. The length of the second sock is found in the same way as that of the first:

$$
\mathrm{L}_{\mathrm{DE}}=\mathrm{L}_{\mathrm{DE}}{ }^{\prime}\left\{10.4 /\left(10.4+\mathrm{h}_{2}\right)\right\}
$$

The height $\mathrm{H}_{\mathrm{E}}$ of the point E at the end of that length is again found by laying the segment $\mathrm{L}_{\mathrm{DE}}$ from D horizontally on your scale drawing of your profile.

Step 6. If you had more socks, you would repeat the procedure used for the second sock for these.

Step 7. Collect and add together the vertical extent of the socks. This is the trickle height:

$$
\mathrm{h}_{\mathrm{t}}=\left\{\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{C}}+\left\{\mathrm{H}_{\mathrm{D}}-\mathrm{H}_{\mathrm{E}}\right\}+\ldots\right. \text { etc. }
$$

Step 8. Record not only $h_{t}$ but also the number of AAV and/ or BPT that you have had to use. These are an inherent parts of your design.

## Notes:

- Negative pressure at one high point may be unavoidable in unusually difficult terrain, but the occurrence of several such points usually means a poorly chosen trench profile. You probably should revisit it. See Appendix B9 for tips about judicious choices.
- To draw a hydraulic grade line with a sock, see Appendix A-II.


## C) You have calculated $h_{\underline{a}}$ and $h_{t}$. You can now evaluate $h_{f_{1}}$ approximately

 as:$$
\mathrm{h}_{\mathrm{f}}=.0053\left\{\mathrm{~L}_{\mathrm{ST}}-\mathrm{L}_{\mathrm{BC}}-\mathrm{L}_{\mathrm{DE}}-\text { etc.... }\right\}
$$

You can now classify your case provisionally:

- If $h_{a} / L>.0053$ you have a case A (or potentially supercritical case).
- If $h_{a} / L<.0030$ you have a case B (or sub-critical case).
- If $0.003<h_{a} / L<0.0053$, you have an intermediate case that can always be transformed into a case B but that may also sometimes be transformed into a case A: See discussion of A2, below.

You can further subdivide case A as follows:

## If $h_{a}>\left(h_{t}+h_{f 1}\right)$, you are dealing with a case A1.



Figure II-2: Case A1

## Case A1

For this category, the head available is sufficient to overcome the sum of the friction at critical flow rate and of the maximum possible trickle height so the flow is able to accelerate into supercritical territory where the head loss due to trickle height disappears. For type A1 flows, you need to do nothing further to deal with air, no matter how many high points without valves you encounter. Your design then consists in the following:

Find the combination of pipe diameters and lengths that will give you a total friction head loss $h_{f}=h_{a}$ at your maximum desired flow rate. This is done as follows:
In the relatively rare cases when in the course of your trickle height calculation you have been forced to provide automatic air valves to prevent negative pressures, you should equip the section in the immediate neighborhood of the valve with a pipe section sufficient, i.e.:

$$
\mathrm{d}>0.933 \mathrm{Q}_{\max }^{2 / 5}
$$

to insure subcritical flow rate at the valve, (see appendix AIII for details).This is judged necessary to ensure that the automatic valve does not open and close abruptly and continuously with the rapid passage of air pockets past it. You will then calculate
the head lost over that section of subcritical flow with the help of the tables or the equations provided in Appendix A.
Over the whole length (from the high point to the following low point) of the remaining socks (those that have not been provided with automatic air valves), you will choose pipe diameters from among those available to you, such that

$$
\mathrm{d}<0.835 \mathrm{Q}_{\max }^{2 / 5}
$$

This diameter, according to (II-1) will be small enough to maintain supercritical flow. You then calculate the head loss due to those sections at maximum flow rate. Finally, you determine from the tables the diameter of the remaining length of pipe, using as $h_{f}, h_{a}$ - (the sum of the two head losses determined above). The relevant flow rate is of course $\mathrm{Q}_{\text {max }}$.

You may ask why it is that the unvented sock areas were treated separately from the rest of the pipe since our classification told us the flow was supercritical over the whole line. This comes from the fact that only finite diameters are available to you so that you may have to split the length of the pipe into segments of two different diameters. When the case is only slightly supercritical, it may happen that the larger diameter flow rate is subcritical. That diameter should then be reserved for the part of the pipe that runs full.

In this case A1, the flow rate through the conduction line will be equal to the output of the spring, whatever that is, up to the flow rate you have chosen as a maximum. In other words, in Case A1, you need not to worry about air in the pipeline either as you start by filling them or during later operations no matter how much the output of the spring varies.

Case A2: $h_{\mathrm{f} 1}<\mathrm{h}_{\mathrm{a}}<\left(\mathrm{h}_{\mathrm{f} 1}+\mathrm{h}_{\mathrm{t}}\right)$. In this case (Figure II-3): if you manage to reach supercritical flow, you will be able to get rid of the air pockets but:

You may well accumulate enough air to prevent you from accelerating to supercritical flow.

Also, remember that in the dry season, the spring output may limit you to a subcritical flow rate in which case an unknown amount of air will stay in the pipes, quite possibly enough to prevent any flow.

You then have the choice of two types of solutions:

1. If you know without question that the flow rate of the spring never falls below a given value, $\mathrm{Q}_{\text {min }}$, you can provide along possible air pockets a pipe diameter sufficiently small for the flow through these sections to be supercritical at Qmin. You will also have to provide manual valves at high points to bleed air when you start the system in order to reach supercritical velocities. The supercritical pipe diameter needs to be used all the way from the high points down to the next low points. You then need to calculate the head lost through these supercritical sections at $\mathrm{Q}_{\text {max. }}$. You may find that this head loss is already larger than the head available. If this is the case, the solution is not satisfactory. If the head loss through the supercritical section at $\mathrm{Q}_{\text {max }}$ is less than the head available, the difference is the head available for friction loss through the rest of the pipe whose diameter you calculate accordingly.
2. Alternatively you may provide enough automatic air valves (together with subcritical sections in their neighborhood) to lower the trickle height $h_{t}$ enough so that $h_{a}>h_{f 1}$. In effect, you reduce the case A2 to a case A1. You may have to experiment a bit with your choice of which high points should get the air valves so as to effect a sufficient trickle height reduction with the minimum number of air valves.


Figure II-3: Case A2
Case B: $h_{a}<h_{f 1}$


Figure II-4. A case B with some flow without air valves
For cases B, head loss from pipe friction alone at critical velocity exceeds the available head. In this case, without air valves, some initial flow may or may not occur depending on the value of $h_{t}$ but the solution is the same in either case. Case B is one for which one has to accept to operate subcritically. Since air pockets cannot be chased out, each air pocket translates into a head loss which in turns means either a completely blocked flow or at least the need to use larger pipe
diameters to compensate for these losses. It is, thus, almost always advisable in case $B$ to provide an automatic air valve at every high point. The pipe sections are chosen so that the head loss by friction at $\mathrm{Q}_{\max }=$ the head available. Whenever Qmax is only slightly subcritial, it is important to ensure that the sections straddling the high points enforce subcritical flow.


Figure II-5 : Another version of case B:
There is no flow without air valves.

## CHAPTER III

## EXAMPLES

The examples which follow have been chosen to illustrate various aspects of the material just presented. Examples requiring longer computations are given in the Guide to the Air-In Pipes software program.

Note: In these examples, for simplicity the full topography of the conduction line is not given. Thus, the elevation of the lower end of an air pocket whose length has been reduced by compression by a given amount is assumed rather than calculated as in the software program.

## Example 1



Figure III-1a

$$
\begin{gathered}
\mathrm{L}_{\mathrm{ST}}=1700 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{SB}}=480 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{BC}^{\prime}}=475 \mathrm{~m} \\
\mathrm{H}_{\mathrm{S}}=31 \mathrm{~m} \quad \mathrm{H}_{\mathrm{A}}=18 \mathrm{~m} \quad \mathrm{H}_{\mathrm{B}}=20 \mathrm{~m} \quad \mathrm{H}_{\mathrm{C}^{\prime}}=0 \quad \mathrm{H}_{\mathrm{T}}=6 \mathrm{~m} \\
\mathrm{Q}_{\max }=0.3 \mathrm{lit} . / \mathrm{sec} .
\end{gathered}
$$

Therefore:

$$
\mathrm{H}_{\mathrm{a}}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}=25 \mathrm{~m} . \mathrm{H}_{\mathrm{SB}}=11 \mathrm{~m} .
$$

There is one local high point at B and so there will be one sock with a trickle height. This high point is higher than the next vented point (Tank T) so that you could if you wished, eliminate the sock by designing a break-pressure tanklet at B. You should always check for this possibility. But in this example, it will turn out that there is no advantage in using it.

Calculate the trickle height. When the pipe upstream of B is full up to S , the static head at B is $\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}=11 \mathrm{~m}$. Therefore,

$$
\mathrm{h}_{1}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}-.0053 \mathrm{~L}_{\mathrm{SB}}=11-.0053(480)=8.46
$$

As a result, the volume of the sock after compression is

$$
10.4 /(10.4+8.46)=0.551
$$

times the volume before compression. If the pipe diameter is uniform along the sock, the length of the sock after compression will also be 0.551 times what it was originally. Assume that after you measure off the length $\mathrm{L}_{\mathrm{BC}}=0.551\left(\mathrm{~L}_{\mathrm{BC}}{ }^{\prime}\right)=$ 262 m (downstream of B ), you find on the pipe profile that the height $\mathrm{H}_{\mathrm{C}}=11 \mathrm{~m}$. Then:

$$
\mathrm{h}_{\mathrm{t}}=\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{C}}=9 \mathrm{~m}
$$

Note that $h_{a}$ is larger than $h_{t}$ so that water will flow when you first fill the pipe.
Next calculate $\underline{h_{f}} \underline{\underline{h_{t}}} \underline{\underline{t}}$ approximately:

$$
\begin{aligned}
& \left.\mathrm{h}_{\mathrm{f} 1}=0.0053 \text { (The length of the part of the pipe that is filled }\right)= \\
& \qquad 0.0053(1700-262)=7.62
\end{aligned}
$$

so that

$$
\mathrm{h}_{\mathrm{f} 1}+\mathrm{h}_{\mathrm{t}}=7.62+9=16.62 \mathrm{~m}
$$

So $h_{a}=25=$ is larger than $h_{f 1}+h_{t}=16.62$. This is CaseA1. (Figure III-1b)
In this case, we use Table 1A to determine $d$ after having determined $h_{f} / L$. We calculate the $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=\left(\mathrm{h}_{\mathrm{a}} / \mathrm{L}_{\mathrm{ST}}\right)$ where $\mathrm{h}_{\mathrm{a}}=25 \mathrm{~m}$, and $\mathrm{L}=\mathrm{L}_{\mathrm{ST}}=1700$ :

$$
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=25 / 1700=0.0147
$$

We enter this value and our maximum flow rate $=0.30$ lit. $/ \mathrm{sec}$. in the table. We find that for a diameter of $1^{\prime \prime}$, and $\mathrm{Q}=0.30 \mathrm{lit} . / \mathrm{sec}$., $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.0091$, (less than the friction we can afford) but for $\mathrm{d}=3 / 4^{\prime \prime}$ and $\mathrm{Q}=0.30 \mathrm{lit} / \mathrm{sec}, \mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.124$ (much more than what we can afford). We could choose $\mathrm{D}=1$ " and check that this will allow us to get almost 0.40 lit./sec. if it turns out that the spring output is that high.

Alternatively (a better practice) we can use a combination of $3 / 4$ " and 1 " pipe lengths to get the system to limit the flow rate to be exactly the maximum flow rate that we were given. You proceed as indicated in Appendix A-IV:

With $L_{1}=$ length of $1^{\prime \prime}$ pipe and $L_{S}=$ length of $3 / 4 "$ pipe in the formula of the appendix, and $\left(h_{f} / L\right)_{S}=0.124$ (the value for $\mathrm{d}=3 / 4$ " and $\mathrm{Q}=0.30 \mathrm{lit} . / \mathrm{sec}$, taken from the table) while $\left(\mathrm{h}_{\mathrm{f}} / \mathrm{L}\right)_{1}=0.00909$ (the value for $\mathrm{d}=1^{\prime \prime}$ and the same Q ):

$$
\begin{gathered}
\mathrm{L}_{1 "}=\{(0.124 \times 1700)-25\} /\{0.124-0.00909\}=1617 \mathrm{~m} \\
\mathrm{~L}_{3 / 4}=1700 \mathrm{~m}-1617 \mathrm{~m}=83 \mathrm{~m} .
\end{gathered}
$$

Now we check our calculation by finding the total head loss for maximum flow which should match the head available:

$$
\mathrm{h}_{\mathrm{f}}=(83 \mathrm{~m} \times 0.124)+(1617 \mathrm{~m} \times 0.00909)=24.99 \mathrm{~m}
$$

which matches our $h_{a}=25 \mathrm{~m}$. The 83 m of smaller pipe diameter is best placed downstream of $\mathrm{C}^{\prime}$ to keep the hydraulic grade line sufficiently high up to point B .

## To summarize:

The system is now designed so that it will deliver without fail any amount of water supplied by the spring up to a maximum of $0.30 \mathrm{lit} . / \mathrm{sec}$. It is not necessary either to provide an automatic air valve at B or to bleed the pipes of air as you start the water going in the empty pipes.

## Example 2

The profile is given below.


Figure III-2
The data are:

$$
\begin{gathered}
\mathrm{H}_{\mathrm{S}}=121 \mathrm{~m} \quad \mathrm{H}_{\mathrm{T}}=93 \quad \mathrm{H}_{\mathrm{A}}=85 \mathrm{~m} \quad \mathrm{H}_{\mathrm{B}}=91 \mathrm{~m} \quad \mathrm{H}_{\mathrm{C}^{\prime}}=0 \\
\mathrm{H}_{\mathrm{D}}=89 \mathrm{~m} \quad \mathrm{H}_{\mathrm{E}^{\prime}}=21 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{ST}}=1150 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{SB}}=150 \mathrm{~m} \\
\mathrm{~L}_{\mathrm{C}^{\prime} \mathrm{D}}=265 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{BC}}=383 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{DE}^{\prime}}=241 \mathrm{~m} \\
\mathrm{Q}_{\min }=0.13 \mathrm{lit} . / \mathrm{sec} \quad \mathrm{Q}_{\max }=0.260 \mathrm{lit} . / \mathrm{s}
\end{gathered}
$$

From which we calculate:

$$
\mathrm{H}_{\mathrm{a}}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}=28 \mathrm{~m} \quad \mathrm{H}_{\mathrm{BC}^{\prime}}=91 \mathrm{~m} \quad \mathrm{H}_{\mathrm{DE}^{\prime}}=68 \mathrm{~m}
$$

We first note that there are two local high points and that we cannot eliminate either one of them with a venting (break pressure) tanklet since they are both lower than $T$. We proceed to calculate the trickle height $h_{t}$. For the sock between $B$ and $C^{\prime}$ the head at $B$ is

$$
\mathrm{h}_{1}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}-(.0053 \times 150 \mathrm{~m})=29.21 \mathrm{~m}
$$

Therefore, according to Appendix A-2.

$$
\mathrm{L}_{\mathrm{BC}} / \mathrm{L}_{\mathrm{BC}^{\prime}}=10.4 /\{10.4+29.21\}=0.263
$$

so that $\mathrm{L}_{\mathrm{BC}}=383 \times .263=100.7 \mathrm{~m}$. On the profile, the point 100.7 m downstream of $B$ is found to be at a height $H_{C}=67 \mathrm{~m}$ so that $\mathrm{H}_{\mathrm{BC}}=91-67=24 \mathrm{~m}$.
We will now need $\mathrm{L}_{\mathrm{CD}}$ :

$$
\mathrm{L}_{\mathrm{CD}}=\mathrm{L}_{\mathrm{CC}^{\prime}}+\mathrm{L}_{\mathrm{C}^{\prime} \mathrm{D}}=\mathrm{L}_{\mathrm{BC}^{\prime}}-\mathrm{L}_{\mathrm{BC}}+\mathrm{L}_{\mathrm{C}^{\prime} \mathrm{D}}=383-100.7+265=547 \mathrm{~m}
$$

We now calculate the head a the next high point D .

$$
\mathrm{h}_{2}=\mathrm{h}_{1}+\mathrm{H}_{\mathrm{C}}-\mathrm{H}_{\mathrm{D}}-.0053 \mathrm{~L}_{\mathrm{CD}}=29.1+67-89-(.0053 \times 547)=4.2 \mathrm{~m}
$$

So the volume compression ratio in the second sock is:

Therefore,

$$
\mathrm{L}_{\mathrm{DE}}=241 \times 0.712=172 \mathrm{~m} .
$$

On the profile, the point 174 m downstream of D is found at a height $\mathrm{H}_{\mathrm{E}}=30 \mathrm{~m}$ so that $\mathrm{H}_{\mathrm{DE}}=38 \mathrm{~m}$. The trickle height is therefore

$$
\mathrm{h}_{\mathrm{t}}=\mathrm{H}_{\mathrm{BC}}+\mathrm{H}_{\mathrm{DE}}=24+38=62 \mathrm{~m}
$$

We note that:

$$
h_{a}=28 \text { while }^{h_{t}}=62
$$

so that:

$$
h_{\mathrm{f}}<\mathrm{h}_{\mathrm{a}}<\mathrm{h}_{\mathrm{t}}+\mathrm{h}_{\mathrm{a}}
$$

Our case here is therefore a Case A2 (See figure III-2b above).
We, therefore, assess the relative merits of the two solutions described in Chapter II.
a) The supercritical solution

We first select the pipe diameter to be located between the high points and the following low points so that $\mathrm{Q}_{\text {min }}$ will remain supercritical there. From equation 2 b, we find that a $3 / 4$ " pipe has a critical flow rate $\mathrm{Q}_{\mathrm{c}}=0.127$ lit./s, slightly less than our $\mathrm{Q}_{\min }$ and therefore (if we trust our Qmin ), suitable in principle. We then evaluate the head loss at $\mathrm{Q}_{\max }$ through the required pipe sections. Their total length is:

$$
\mathrm{L}_{\mathrm{BC}^{\prime}}+\mathrm{L}_{\mathrm{DE}^{\prime}}=383 \mathrm{~m}+241 \mathrm{~m}=624 \mathrm{~m}
$$

And with $\mathrm{d}=0.0231 \mathrm{~m}$ and $\mathrm{Q}_{\max }=0.260 \mathrm{l} / \mathrm{sec}$, from the Tables of Appendix 1, approximately $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=.025$ (more precisely .0 .0251 ). So the head loss in the imposed supercritical sections is:

$$
0.025 \times 624 \mathrm{~m}=15.6 \mathrm{~m}
$$

This leaves us: $\mathrm{h}_{\mathrm{a}}-15.6=28-15.6=12.4 \mathrm{~m}$ of head loss for the rest of the line whose length is $L_{S T}-624=1150 \mathrm{~m}-624=526 \mathrm{~m}$. The allowed head loss $/ \mathrm{m}$ is then :

$$
12.4 / 526=.0236
$$

which, as we have just seen is slightly less than that of a $3 / 4$ " pipe at $\mathrm{Q}_{\text {max }}$. We then use Appendix A-III (equations A-4 \& A5) to find the right combination of lengths of pipes of $3 / 4$ " and 1 " over the 526 m of length to get precisely 12.4 m of head loss at $\mathrm{Q}_{\max }$. At $\mathrm{Q}_{\max }=0.2601 / \mathrm{sec}, \mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.025$ for $\mathrm{d}=.0231$ and $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.0070$. Therefore,

$$
\begin{gathered}
\mathrm{L}_{1^{\prime \prime}}=\{526(.025)-12.4\} /(0.025-.0070)=41.7 \mathrm{~m} \\
\mathrm{~L}_{3 / 4 \prime \prime}=526-41.7=484.3 \mathrm{~m}
\end{gathered}
$$

As a matter of routine, the larger 1" section should be located early- immediately downstream of the spring.

Critique: This is a good solution provided:

- You don't forget to place manual valves at the two high points to drain the air when you initiate the flow.
- You are absolutely sure that $\mathrm{Q}_{\underline{\min }}$ will not fall below your estimate.
b) The conversion to a case A1 by means of a single automatic valve.

It is not immediately clear whether we should place it at the first or at the second high point. Say we start with the first:

The pressure at D is

$$
\left\{\mathrm{H}_{\mathrm{s}}-\mathrm{H}_{\mathrm{d}}-.0053(150+383+265\}=27.8 \mathrm{~m}\right.
$$

so that

$$
\mathrm{L}_{\mathrm{DE}} / \mathrm{L}_{\mathrm{DE}}=10.4 /(10.4+27.8)=0.27 \text { and } \mathrm{L}_{\mathrm{DE}}=65 \mathrm{~m}
$$

On the pipe profile, this puts the point E , say, 13.6 m below D . This is then the new trickle height $h_{t}$. We note that $h_{t}<h_{a}$ so that we have successfully converted to a case A1.
We also note that if we had placed the automatic valve at the second high point D , the trickle height would have been (from previously) 24 m , which would have permitted the conversion to case A1, but less comfortably so- hence we would choose to place the valve at $B$.

In case you are not sure what your $\mathrm{Q}_{\text {mi. }}$ is, solution b ) should be adopted.

Example 3: What if you don't worry about air?
The profile is sketched on figure III-3a. The geometric data is:


Figure III-3

$$
\begin{gathered}
\mathrm{H}_{\mathrm{S}}=48 \mathrm{~m} \quad \mathrm{H}_{\mathrm{C}^{\prime}}=0 \quad \mathrm{H}_{\mathrm{A}}=36 \mathrm{~m} \quad \mathrm{H}_{\mathrm{B}}=38 \mathrm{~m} \quad \mathrm{H}_{\mathrm{T}}=41 \mathrm{~m} ; \quad \mathrm{L}_{\mathrm{ST}}=1100 \mathrm{~m} \\
\mathrm{~L}_{\mathrm{SB}}=145 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{BC}^{\prime}}=267 \mathrm{~m}
\end{gathered}
$$

The available and desired flow rate is estimated as 15 lit./min.
Here we first ignore what we have learned above and design without paying attention to the possibility of air trouble.

Standard Friction Design. The available head, $\mathrm{h}_{\mathrm{a}}$ is:

$$
\mathrm{h}_{\mathrm{a}}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}=48-41=7 \mathrm{~m}
$$

We calculate the average slope, $\mathrm{h}_{\mathrm{a}} / \mathrm{L}$ :

$$
\mathrm{h}_{\mathrm{a}} / \mathrm{L}=7 / 1100=0.00636
$$

This is very close to the friction slope for the desired flow rate ( $0.25 \mathrm{l} / \mathrm{sec}$.) for a 1" PVC pipe, SDR 26 (see Table A1 or carry out the calculations in Appendix A for a pipe with diameter $=0.0300 \mathrm{~m}$ ). If we use a $1^{\prime \prime}$ pipe from $S$ to $T$ and if the
pipe is full of water, we should be able to get the flow rate that we desire.
Furthermore the maximum pressure head is about 43 m (well within the rating of SDR 26) and there is no negative pressure anywhere so a single diameter pipe seems appropriate.

How well will it work? Now, we worry about starting the system and about what happens if the output of the spring falls below $15 \mathrm{l} / \mathrm{min}$.

We note first from the profile (only one local maximum), that only one air sock is possible. We notice also that we cannot use a simple break-pressure tank at B, since $B$ is lower than $T$. We calculate the trickle height $h_{t}$, i.e. the height of the air sock, when the water in the pipe backs up to the level of S. In this case, the pressure head at $B$ is

$$
\mathrm{h}_{1}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}-.0053 \mathrm{~L}_{\mathrm{SB}}=9.23 \mathrm{~m}
$$

So

$$
\mathrm{L}_{\mathrm{BC}^{\prime} / \mathrm{L}_{\mathrm{BC}}=10.4 /\{10.4+9.2\}=.531}
$$

Then,

$$
\mathrm{L}_{\mathrm{BC}}=267(0.531)=142 \mathrm{~m} \text { and } \mathrm{L}_{\mathrm{SC}}=145+142=387 \mathrm{~m}
$$

Here, on the scale drawing of the pipe, measure off the distance BC of the sock once compressed and read the height of point $C$. Suppose that this gives $H_{C}=19 \mathrm{~m}$ so that

$$
\mathrm{h}_{\mathrm{t}}=\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{C}}=38-19=19 \mathrm{~m}
$$

This is a large potential trickle height since $h_{a}$ is only 7 m . And this raises two problems:

Starting: Now, since $h_{t}$ is larger than $h_{a}$, you will probably have to bleed the air out of the sock before any water will flow out at T. You can do that with a T or a valve just downstream of B. If you bleed all the air out, the system will deliver the design flow rate as long as it is provided by the spring.

Sustained Operation: Now, suppose that the spring output decreases during the dry season. In other words, suppose for instance that it is or becomes only 13.5 lit./min. (or less). This is a very small decrease- $90 \%$ of the estimated flow rate. However, this amount is also less than the critical flow rate, $\mathrm{Q}_{\mathrm{S}}$ for this size of pipe since $\mathrm{Q}_{\mathrm{S}}=14.7 \mathrm{l} / \mathrm{min}$. for a $1^{\prime \prime}$ pipe (see Table A3, Appendix A). With a spring flow rate less than both the critical flow rate and the flow rate that your chosen pipe friction calls for, air in the form of small or medium size bubbles will travel from the spring exit downstream and keep accumulating in the sock. This is serious. For instance, by the time the sock height is $1 / 3$ of its maximum height, you would only have 0.65 m of head left to overcome pipe friction which would reduce your flow to $0.018 \mathrm{lit} . / \mathrm{s}=1 \mathrm{lit} / \mathrm{min}$. instead of 15 . In fact, if the decrease in the spring output occurs fairly rapidly, enough air will accumulate in the sock to block the flow completely.

Cure (Standard): You may install a float-type air purging valve downstream of B. In general this is not a good idea for flow rates which are supercritical, as explained in Chapter II because these flows tend to cause the valve mechanism to turn on and off rapidly and all the time- a source of wear. A better idea is to:

## Follow the procedure recommended in this manual!

The first step since we have calculated $h_{t}$ is to classify this case. Following the procedure outlined in Chapter II and used in the first example we calculate $\mathrm{h}_{\mathrm{f} 1}$ :

$$
\mathrm{h}_{\mathrm{f} 1}=0.0053(1100-142)=5.1 \mathrm{~m}
$$

so that

$$
\mathrm{h}_{\mathrm{fl}}<\mathrm{h}_{\mathrm{a}}<\mathrm{h}_{\mathrm{fl}}+\mathrm{h}_{\mathrm{t}}
$$

We recognize that this is a Case A2 although very close to a case B (see Figure III-3). The figure makes it clear that, as we have already found out, a very small decrease in flow rate out of the spring leads to the possibility of an air sock. Besides, we have not been given a minimum flow rate out of the spring so that we cannot follow the option offered in Chapter II for this case which suggests that you set $\mathrm{Q}_{\mathrm{c}}<\mathrm{Q}_{\min .}$. We don't even know the value of $\mathrm{Q}_{\max }$ ! This is life! We do our best:
As explained in the section n Chapter II dealing with a case A2, we could choose a section guaranteeing supercritical flow in the sock region for a flow rate lower than the prescribed $11 / \mathrm{min}$. We already know that a 1 " section does not leave us enough margin. A $3 / 4$ " (SDR17 or $\mathrm{d}=.0231$ ) section would according to equation impose supercritical flow down to:

$$
\mathrm{Q}=1.57 * 0.0231^{2.5}=1.27 \mathrm{E}-4 \mathrm{~m}^{3} / \mathrm{sec}=7.61 / \mathrm{min}
$$

Or roughly half the desired flow rate. We don't know whether that is adequate, since we have no knowledge of minimum flow. If you judge this safe, (i.e. if you are sure- but don't ask me how -the flow rate won't fall below that value), you proceed as follows:
You reserve the $3 / 4 "$ section between B an C' (note: not C) ,i.e. for 267 m . This consumes at the desired flow rate of $0.25 \mathrm{l} / \mathrm{sec}$ (see Table A1) $0.0229 \times 267=6.11 \mathrm{~m}$. That only leaves

$$
\mathrm{H}_{\mathrm{a}}-6.11=7-6.11=0.89 \mathrm{~m} \text { of head }
$$

for the remaining $1100 \mathrm{~m}-267 \mathrm{~m}=833 \mathrm{~m}$.So for that section

$$
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=.89 / 833=0.00107
$$

which according to the table, is almost exactly accommodated by a 1.5 " pipe.

## To summarize:

1100 m of 1" pipe have approximately the same friction at 15 lit./s. 269 m of $3 / 4$ " and 831 m of $1.5^{\prime \prime}$ pipe so that the new combination will give the same flow rate when the pipe is full as the original design. But the new design will not get into air trouble until the flow rate has fallen below $51 \%$ of the expected flow rate provided the smaller pipe is positioned where the air socks might form.

You would still need to bleed the pipe as you fill it initially, of course, and for this, you could use a simple T which you could cap after the start. The 1.5 " section, you would locate anywhere except between B and $\mathrm{C}^{\prime}$.

The additional cost of the modification is substantial.

## Alternate (and preferred) solution

In this particular case (no knowledge of minimum flow), it is safer to use an automatic valve at the high point and to force subcritical flow at the beginning of the sock section. For the maximum flow $=151 / \mathrm{min}=0.251 / \mathrm{sec}$, Equation 2a gives:

$$
\mathrm{d}>(\mathrm{Q} / 1,19)^{2 / 5}=0.0338 \mathrm{~m} .
$$

So that a $11 / 4$ " pipe (SDR $32.5, \mathrm{~d}=.0391$ ) would be adequate. It would be placed somewhat earlier than the high point and would trail the automatic valve by a short distance. The automatic valve would be placed sufficiently downstream of the high point to be sure that a mistake in the location of this high point would not place the valve upstream of that point (which would render the valve useless). Recommendations for putting these precautions into effect are found in Appendix A-III.

Let us assume that following them the length of the $1 \& 1 / 4$ " section is found to be 70 meters. The head lost through that section at maximum flow rate is then, (equation A2a):

$$
\mathrm{h}_{\mathrm{f}}=70\left(7.76 \mathrm{E}^{-4}\right)(2.5 \mathrm{E}-4)^{7 / 4} / .0391^{19 / 4}=0.13 \mathrm{~m}
$$

The remaining head available for the rest of the pipe $(1100-70=1030 \mathrm{~m})$ is 7 $0.13=6.87 \mathrm{~m}$ or $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=.00667$ which is almost exactly the head loss of a 1 " pipe, SDR26.

This solution is not only safer but also cheaper (no need for a lot of 1.5 " pipe) but it requires an automatic air valve.

## Example 4

This example is adapted from a case near Quolga Khoya, in the Cochabamba region of Bolivia. A community of 16 families has access to water for washing and cattle from a canal but this is not suitable for drinking. About 1 km away from the village and 4 m above a possible distribution tank is a small spring through broken
rock on a steep slope. An unlined earth canal with a very small slope used to bring the water to the village (presumably when it was more abundant). The campesinos have now dug the canal deeper and intend to place a PVC pipe at the bottom of the trench and bury it to protect the small water supply and prevent seepage losses.

The spring output is about 1.5 lit./min. Will the system work?
This example illustrates one important practical consideration in the construction of gravity flow systems: It is easy to introduce unintentionally local high points which create unexpected trickle height losses. This is a particular danger when the desired profile is almost horizontal at sections where the head is small. In this case, the high points may even be accidentally higher than the spring level.

For the present case, we first calculate the friction head required. Even with the smallest easily available PVC pipe diameter $\mathrm{d}=1 / 2^{\prime \prime}$, this friction loss is small.

For this very small flow rate, Table A1 gives approximately for $\mathrm{d}=0.0173 \mathrm{~m}, \mathrm{~h}_{\mathrm{f}} / \mathrm{L}$ $=0.0016$ or $\mathrm{h}_{\mathrm{f}}=1.6 \mathrm{~m}$, so that the available head, 4 m , is more than adequate for the friction head loss. But even for $\mathrm{d}=1 / 2^{\prime \prime}$, the flow will always be subcritical (see table A-3). Since the trench follows a canal, there are no intended local high points in the profile. But normally as the workers lay and bury the pipe at the bottom, it will not be perfectly flat. This is particularly true of polyethylene pipes which are manufactured in rolls. But even with PVC, the pipe can be expected to go up and down an inch or two several times for each pipe section. Each one of these rises, if they exceed the inner pipe diameter ( $1 / 2{ }^{\circ}$ ) will cause a small trickle height and if the cumulative total of these small head losses exceed 4 m , there can be no flow at all through the pipe. The remedy is time-consuming because the workers need to insure that everywhere the pipe as laid has a downstream slope. This needs to be done with a carpenter's level or by trying the pipeline before the trench is filled. Also, the trench filling needs to be done with great care.

## Example 5

Similarly in the actual construction of the system, small deviation from an intended profile can occasionally cause serious problems. Consider the following intended (III-5a) and actual (III-5b) profiles inspired by a case near Rio-Blanco, Nicaragua:


Figure III-5
$\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{T}}=6 \mathrm{~m} ; \quad \mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}=7 \mathrm{~m} ; \quad \mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{C}^{\prime}}=70 \mathrm{~m} ; \quad \mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{A}}=0.30 \mathrm{~m}$.
$\mathrm{L}=600 \mathrm{~m}$. $\quad \mathrm{Q}$ between 8 and $18 \mathrm{l} / \mathrm{min}$.
In the design, there is no trickle height at all and the pipe diameter has been chosen on the basis of friction losses. The available head allows a $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=6 / 600=0.01$ For a maximum flow rate of 18 lit. $/ \mathrm{min} .=0.3 \mathrm{lit} / \mathrm{s}$., a 1 " pipe $($ SDR 26$)$ has a $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=$ 0.0091. It is, therefore, suitable and if the construction conforms to figure III-5a, all is well. But if the profile turns out to be as in figure III-5b (a very minor change in height), there is a considerable trickle height whose maximum value needs to be computed for a specified profile between B and C' but which is likely to be about 40 meters- much more than the available head. With this trickle height, you can expect complete blocking, a situation which clearly needs to be avoided. In this case, it is far better to get rid of the low point at A than to add a float-type air valve beyond $B$ because the system operates supercritically a good deal of the time ( $\mathrm{Q}_{\mathrm{S}}=14.7$ ). Anyway, always strive for the simplest installation, the one which requires the fewest number of moving parts.

## Example 6

$$
\begin{aligned}
& \mathrm{H}_{\mathrm{S}}=40 \mathrm{~m} \quad \mathrm{H}_{\mathrm{A}}=21 \mathrm{~m} \quad \mathrm{H}_{\mathrm{C}^{\prime}}=0 \quad \mathrm{H}_{\mathrm{T}}=20 \mathrm{~m} \quad \mathrm{H}_{\mathrm{B}}=26 \mathrm{~m} \\
& \mathrm{~L}_{\mathrm{ST}}=4700 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{SB}}=200 \mathrm{~m} \quad \mathrm{~L}_{\mathrm{BC}^{\prime}}=786 \mathrm{~m} \\
& \mathrm{Q}_{\min }=0.10 \text { lit. } / \mathrm{s} . \quad \mathrm{Q}_{\max }=0.217 \text { lit. } / \mathrm{s} .
\end{aligned}
$$



Figure III-6
This is an example for which there are three possible solutions. But one of them, which involves allowing for an unknown amont of air in a case B is not recommended and will ot be retained.

Solution 1: There is one high point (one possible sock) at B. Since B is higher than the tank at T , we can get rid of the sock problem by placing a break-pressure tanklet there. Then, we can accommodate any flow up to the maximum by equating the head available between S and B to the friction head loss between these two points at $\mathrm{Q}_{\max }$ and doing the same for the section BT .
For SB , the head available is $\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}=14 \mathrm{~m}$ and the length is $\mathrm{L}_{\mathrm{SB}}=200 \mathrm{~m}$. So, the maximum friction loss ratio $h_{f} / L=14 / 200=0.070$. Table A1 gives $h_{f} / L$ (by interpolation) $=0.07$ for $\mathrm{Q}=0.2171 / \mathrm{sec}$. and $\mathrm{d}=1 / 2^{\prime \prime}$. We shall, therefore, use $1 / 2^{\prime \prime}$ pipe for this first section.
For the section $B T, L_{B T}=4700 \mathrm{~m}-200 \mathrm{~m}=4500 \mathrm{~m}$. and $\mathrm{H}_{\mathrm{B}}-\mathrm{H}_{\mathrm{T}}=6 \mathrm{~m}$ so the maximum friction ratio allowed for this segment is $6 \mathrm{~m} / 4500 \mathrm{~m}=0.00133$. We should therefore plan for a case B. According to Table A1 (again by interpolation) for a $\mathrm{Q}=0.217$, a $1^{\prime \prime}$ pipe has a $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.0051$ and a $1.5^{\prime \prime}$ pipe has a $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=$ 0.000812 . So, we need a mix of these two diameters. Turning to Section A3, we find that :

$$
\begin{gathered}
\mathrm{L}_{\mathrm{d}}=1.5^{\prime \prime}=\mathrm{L}_{\mathrm{l}}=\{(0.0051(4500)-6\} /(0.0051-0.000812)=3950 \mathrm{~m} \\
\mathrm{L}_{\mathrm{d}=1 "}=\mathrm{L}_{\mathrm{S}}=4500 \mathrm{~m}-3950=550 \mathrm{~m}
\end{gathered}
$$

Check: $\{0.000812(3950)+(0.0051) 550\}=6.001 \mathrm{~m}$
Summary solution 1): From T to B 200m of $1 / 2^{\prime \prime}$ pipe. From B to T first 3950 mm of $1.5 "$ pipe, then 550 m of 1 " pipe. A break pressure unit at B.

## The second solution:

Without a break-pressure unit at B, we have to allow for a sock after B.

- Calculate $h_{t}$ :

$$
\begin{gathered}
\mathrm{h}_{1}=\mathrm{H}_{\mathrm{S}}-\mathrm{H}_{\mathrm{B}}-.0053 \mathrm{~L}_{\mathrm{SB}}=14-(.0053 \times 200)=12.9 \mathrm{~m} \\
\mathrm{~L}_{\mathrm{BC}} / \mathrm{L}_{\mathrm{BC}^{\prime}}=10.4 /(10.4+12.9)=0.446
\end{gathered}
$$

So, $\mathrm{L}_{\mathrm{BC}}=0.446 \times 786 \mathrm{~m}=350 \mathrm{~m}$. On your profile of the pipeline, you find that the height of point $C$ is 14.9 m so that $h_{t}=11.1 \mathrm{~m} ., h_{a}=H_{S}-H_{T}=20 \mathrm{~m}$ and from equation A-4:

$$
\mathrm{h}_{\mathrm{f} 1}=0.0053 \times(4700-350)=23.05 \mathrm{~m}
$$

From this, we get:

$$
\mathrm{h}_{\mathrm{a}} / \mathrm{h}_{\mathrm{t}}=1.80 \quad \mathrm{~h}_{\mathrm{f} 1} / \mathrm{h}_{\mathrm{t}}=2.08
$$

So $\mathrm{h}_{\mathrm{a}}$ is smaller than $\mathrm{h}_{\mathrm{fl}}$. This is a Case B (Figure III-6b: The slope is insufficient to allow supercritical flow). It is true that even with the maximum possible air losses, some water will flow (ha $>h_{t}$ and therefore, $h_{a}>$ head loss for smaller flow rates) but it is a bad idea to design your system without getting rid of the air for two reasons:

1) Since you don't know how much air the system will have at any particular time, you cannot design a known cap on $\mathrm{Q}_{\text {max }}$.
2) To make up for the trickle height loss, you would have to use larger pipe sections with a larger pipe cost.

So, unless you have no access to automatic air valves, you will place one "at" B and design for a simple friction with $\mathrm{Q}_{\text {max }}$.

The friction ratio or slope is $\mathrm{h}_{\mathrm{f}} / \mathrm{L}=\mathrm{h}_{\mathrm{a}} / \mathrm{L}_{\mathrm{ST}}=20 / 4700=0.00426$. At $\mathrm{Q}_{\max }=$ $0.217 \mathrm{l} /$ sec., as we have just seen, $\mathrm{h}_{\mathrm{f}} / \mathrm{L}$ is .0051 for a 1 " pipe and it is 0.000812 for a $1.5 "$ "pipe. The proper combination of these is, following Section A3,

$$
\begin{gathered}
\mathrm{L}_{\mathrm{l}}=\mathrm{L}_{1.5^{\prime \prime}}=\{(0.0051 \times 4700)-20\} /(0.0051-0.000812)=926 \mathrm{~m} \\
\mathrm{~L}_{\mathrm{S}}=\mathrm{L}_{1}{ }^{\prime \prime}=4700-926=3774 \mathrm{~m}
\end{gathered}
$$

Check: $(3774 \times .00510)+(926 \times 0.000812)=19.99 \mathrm{~m}$

Summary of Solution 2: With a air-bleeding float valve "at" B, the right maximum flow rate is achieved with 926 m of $1.5^{\prime \prime}$ pipe and 3774 m of $1^{\prime \prime}$ pipe. The order in which these diameters are chosen is not important in this particular case because the hydraulic grade line will remain above the pipe profile for any order. But one normally uses the larger section pipes upstream..

The results for the two solutions are summarized below:

| diameter | solution 1 | solution 2 |
| :---: | :---: | :---: |
| $1 / 2^{\prime}$ | 200 m | 0 |
| $1^{\prime \prime}$ | 550 m | 3774 m |
| $1.5^{\prime \prime}$ | 3950 m | 926 m |
| Device | Break-pressure <br> tank | Air-bleed <br> valve |

From this table, it appears that solution 2 would be the cheapest: lower pipe cost and the automatic air-bleeding valve installation is generally cheaper than a break pressure tank. But it requires a device with moving parts.

Example 7: For our final example, we use the same profile as for example 6 but we raise the height of the end tank $T$ to 31 m . This only changes $h_{a . .} Q_{\max }=0.15$ $1 / \mathrm{sec} .$. We have:
$\mathrm{h}_{\mathrm{a}}=9 \mathrm{~m} . \quad \mathrm{h}_{\mathrm{t}}=12.9 \mathrm{~m} \quad \mathrm{~h}_{\mathrm{f} 1}=24.71 \mathrm{~m}$
Note that $T$ is higher than $B$ now so that solution 1 of the previous example cannot be used. $h_{a}$ is smaller than $h_{t}$ and also smaller than $h_{f 1}$. This is also a case B, only different from the previous one by the possibility that without an automatic air bleeding valve, no flow at all will result (Figure III-7). This means that we have to use an air bleeding float-valve at B and that we evaluate:

$$
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=\mathrm{h}_{\mathrm{a}} / \mathrm{L}_{\mathrm{ST}} \cdot=9 / 4700=0.00191
$$

For $\mathrm{Q}=\mathrm{Q}_{\max }=0.15 \mathrm{l} / \mathrm{sec}$. Table A 1 gives us:

$$
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=0.00270 \text { for } \mathrm{d}=1 \text { " and } \mathrm{h}_{\mathrm{f}} / \mathrm{L}=.0042 \text { for } \mathrm{d}=1.5^{\prime \prime}
$$

So, according to Section A3:

$$
\begin{gathered}
\mathrm{L}_{1.5^{\prime \prime}}=\mathrm{L}_{\mathrm{l}}=\{(0.00270 \times 47000)-9\} /(0.00270-0.00042)=1618 \mathrm{~m} \\
\mathrm{~L}_{1 \prime}=\mathrm{L}_{\mathrm{S}}=4700-1618=3082 \mathrm{~m}
\end{gathered}
$$

Check: $1618(0.00042)+3082(0.00270)=9 \mathrm{~m}$


Figure III-7

## APPENDIX A

## A-1: Summary of Formulas and Tables for Conventional

## ( FULL PIPE) Hydraulic Calculations in Pipes.

These apply if there are no stationary air socks. Stationary air socks will not occur if:

- There is no unvented local maximum in the pipeline profile.
- Or the pipe runs full.
- Or the pipe runs with a flow rate Q greater than the critical flow rate $\mathrm{Q}_{\mathrm{c}}$.

One exception: If the hydraulic grade line is locally sufficiently below the pipe line elevation to cause cavitations ( 8 to 9 meters), there will be not air but water vapor there and also a great deal of shocks and knocks caused by the collapse and reforming of vapor bubbles. This will damage the pipe and should be avoided.

The basic equation for a single pipe (without branching) between an upstream point 1 and a downstream point 2 is:

$$
.0826 \mathrm{Q}^{2} / \mathrm{d}_{1}^{4}+\mathrm{h}_{1}+\mathrm{H}_{1}=0.0826 \mathrm{Q}^{2} / \mathrm{d}_{2}^{4}+\mathrm{h}_{2}+\mathrm{H}_{2}+\mathrm{h}_{\mathrm{f} 12(\mathrm{~A} 1)}
$$

In this equation, $h$ is the pressure head in units of meters (the pressure head is the pressure above atmospheric pressure divided by the weight of a m 3 of water); H is the height of the pipe line at any point (with respect to any fixed datum) and $\mathrm{h}_{\mathrm{f} 12}$ is the friction head loss between any two points 1 and 2 .

A hydraulic grade line (HGL) is a line which plots the height of the sum $(\mathrm{h}+\mathrm{H})$ as a function of position along the pipe. One usually plots also the height H of the pipeline on the same graph. In principle, there should be one more line, the Energy line whose height represents the sum of all three terms in the equation. But for drinking water designs, the first term on the left of either side of the equation is quite small compared to the others so that the hydraulic grade line and the energy line are almost the same. This is because the recommended velocity in drinking water systems does not exceed $3 \mathrm{~m} / \mathrm{s}$. This gives a maximum difference in levels between the energy and hydraulic grade line of less than 50 cms . For pipelines leading to pumps and turbines, this is far from true and the energy line should always be shown.

Since the height of the hydraulic grade line is $(\mathrm{H}+\mathrm{h})$, while the height of the pipe is H , the vertical spacing between the hydraulic grade line and the pipe height is the
pressure head h . If the HGL falls below the pipeline height, the pressure head is negative (i.e. less than atmospheric ).

Wherever the pipe is vented (spring, break pressure tank, distribution tank, valve opened to the atmosphere), the HGL and the pipe profile have the same height.

In the absence of a pump, the energy line and so also the HGL in our case, always decrease downstream because $\mathrm{h}_{\mathrm{f}}$ is always a loss of head.

Friction Losses: The equation you can use, instead of TableA1 to calculate $h_{f}$ or to determine the hydraulic grade line, given $d \& Q$ and in the absence of air socks is due to Blasius ${ }^{1}$. It is:

$$
\begin{equation*}
\mathrm{h}_{\mathrm{f}} / \mathrm{L}=7.76 \times 10^{-4} \mathrm{Q}^{7 / 4} / \mathrm{d}^{19 / 4} \tag{A2a}
\end{equation*}
$$

where he units of Q are $\mathrm{m}^{3} / \mathrm{sec}$ and those of d are meters.
If you know $Q$ and $h_{a}$ instead, you get $d$ from:

$$
\begin{equation*}
d=0.222 \mathrm{Q}^{7 / 19}\left(\mathrm{hf}_{\mathrm{f}} / \mathrm{L}\right)^{-4 / 19} \tag{A2b}
\end{equation*}
$$

And if you know $h_{a}$ and $d$, you can get $Q$ from:

$$
\begin{equation*}
\mathrm{Q}=59.9\left(\mathrm{~h}_{\mathrm{f}} / \mathrm{L}\right)^{4 / 7} \mathrm{~d}^{19 / 7} \tag{A2c}
\end{equation*}
$$

Equation (A2a) was used to calculate $\mathrm{h}_{\mathrm{f}} / \mathrm{L}$ in the tables A1 which are found on the next pages.

Equations 3 and 4 of Chapter II, the approximate equations used to classify your cases are reproduced here for convenience:

$$
\begin{array}{r}
\mathrm{h}_{\mathrm{f}} / \mathrm{h}_{\mathrm{t}}=0.0053 \mathrm{Q}^{* 2}\left(\mathrm{~L} / \mathrm{h}_{\mathrm{t}}\right) \\
\mathrm{h}_{\mathrm{f} 1} / \mathrm{h}_{\mathrm{t}}=0.00568 \mathrm{~L} / \mathrm{h}_{\mathrm{t}} \tag{A4}
\end{array}
$$

The length L here does not include the length of the socks.
Note: For the following tables, the inner diameters assumed are as follows:

| Nominal diam. | SDR\# | Diam, m. |
| :---: | :---: | :---: |
| $1 / 2$ | 13 | 0.0173 m |
| $3 / 4^{\prime \prime}$ | 17 | 0.0231 m |
| $1 "$ | 26 | 0.0300 m |
| $1.5^{\prime \prime}$ | 26 | 0.0444 m |
| $2.0^{\prime \prime}$ | 26 | 0.0557 m |
| $2.5^{\prime \prime}$ | 26 | 0.0674 m |

The diameters assumed in all the examples of Chapter III are these.

[^1]Table A 1 was prepared by using Equation A-2. It can be used instead of A-2.
TABLE A-1: FRICTION HEAD LOSSES

| $\mathrm{d}=1 / 2^{\prime \prime}=\mathbf{0 . 0 1 7 3 m}$ |  | d=3/4" $=0.0231 \mathrm{~m}$ |  | $\mathrm{d}=1^{\prime \prime}=0.0300 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q, l/sec | $h_{f} / \mathrm{L}$ | Q, 1/sec | $h_{f} / \mathrm{L}$ | Q, 1/sec | $h_{f} / \mathrm{L}$ |
| 0.010 | . 000323 | 0.01 | . 000082 | 0.03 | . 00016 |
| 0.015 | . 000657 | 0.02 | . 000275 | 0.05 | . 00040 |
| 0.020 | . 00109 | 0.03 | . 00056 | 0.075 | . 00080 |
| 0.025 | . 00161 | 0.05 | . 00137 | 0.10 | . 00133 |
| 0.030 | . 00221 | 0.075 | . 00278 | 0.125 | . 00196 |
| 0.040 | . 00365 | 0.10 | . 00460 | 0.150 | . 00270 |
| 0.050 | . 00540 | 0.125 | . 00680 | 0.175 | . 00353 |
| 0.075 | . 0101 | 0.150 | . 00936 | 0.200 | . 00447 |
| 0.100 | . 0182 | 0.175 | . 0122 | 0.225 | . 00550 |
| 0.125 | . 0268 | 0.20 | . 0158 | 0.250 | . 00661 |
| 0.150 | . 0309 | 0.225 | . 0190 | 0.275 | . 00780 |
| 0.175 | . 0483 | 0.250 | . 0229 | 0.300 | . 00909 |
| 0.200 | . 0611 | 0.275 | . 0270 | 0.325 | . 0105 |
| 0.225 | . 0751 | 0.300 | . 0315 | 0.350 | . 0119 |
| 0.250 | . 0903 | 0.325 | . 0362 | 0.375 | . 0134 |
| 0.275 | . 106 | 0.350 | . 0412 | 0.400 | . 0150 |
| 0.300 | . 124 | 0.375 | . 0465 | 0.425 | . 0167 |
| 0.325 | . 143 | 0.400 | . 0521 | 0.450 | . 0185 |
| 0.350 | . 163 | 0.425 | . 0579 | 0.475 | . 0203 |
| 0.375 | . 184 | 0.450 | . 0640 | 0.500 | . 0222 |
| 0.400 | . 205 | 0.475 | . 0703 | 0.525 | . 0242 |
| 0.425 | . 228 | 0.500 | . 0769 | 0.550 | . 0263 |
| 0.450 | . 252 | 0.525 | . 0837 | 0.575 | . 0284 |
| 0.475 | . 278 | 0.550 | . 0909 | 0.600 | . 0306 |
| 0.500 | . 303 | 0.575 | . 0982 | 0.625 | . 0328 |
| 0.525 | . 330 | 0.600 | . 106 | 0.650 | . 0352 |
| 0.550 | . 359 | 0.625 | . 114 | 0.675 | . 0376 |
| 0.575 | . 388 | 0.650 | . 122 | 0.700 | . 0400 |
| 0.600 | . 418 | 0.675 | . 130 | 0.725 | . 0426 |
|  |  | 0.700 | . 139 | 0.750 | . 0452 |
|  |  | 0.725 | . 147 | 0.775 | . 0478 |
|  |  | 0.750 | . 156 | 0.800 | . 0506 |
|  |  | 0.800 | . 175 | 0.825 | . 0534 |
|  |  | 0.825 | . 185 | 0.850 | . 0562 |
|  |  | 0.850 | . 195 | 0.875 | . 0592 |

TABLE A-1: FRICTION HEAD LOSSES (continued)

| $\mathrm{d}=1{ }^{\prime \prime}=0.0300 \mathrm{~m}$ (cont.) |  | $\mathrm{d}=1.5^{\prime \prime}=0.0444 \mathrm{~m}$ |  | $\mathrm{d}=1.5^{\prime \prime}=0.0444 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q, 1/sec | $\mathrm{hf}_{\mathbf{f}} / \mathrm{L}$ | Q, 1/sec | $h_{f} / \mathrm{L}$ | Q, 1/sec | $h_{f} / \mathrm{L}$ |
| 0.900 | . 0622 | 0.10 | . 00021 | 1.90 | . 0357 |
| 0.925 | . 0652 | 0.15 | . 00042 | 1.95 | . 0374 |
| 0.950 | . 0684 | 0.20 | . 00069 | 2.00 | . 0390 |
| 0.975 | . 0715 | 0.25 | . 00102 | 2.05 | . 0408 |
| 1.000 | . 0748 | 0.30 | . 00141 | 2.10 | . 0425 |
| 1.025 | . 0780 | 0.35 | . 00185 | 2.15 | . 0443 |
| 1.050 | . 0814 | 0.40 | . 00234 | 2.20 | . 0461 |
| 1.075 | . 0848 | 0.45 | . 00287 | 2.30 | . 0499 |
| 1.10 | . 0883 | 0.50 | . 00345 | 2.40 | . 0537 |
| 1.125 | . 0918 | 0.55 | . 00408 | 2.50 | . 0577 |
| 1.15 | . 0954 | 0.60 | . 00475 | 2.60 | . 0618 |
| 1.175 | . 0991 | 0.65 | . 00546 | 2.70 | . 0660 |
| 1.20 | . 103 | 0.70 | . 00621 | 2.80 | . 0704 |
| 1.25 | . 110 | 0.75 | . 00702 | 2.90 | . 0748 |
| 1.30 | . 118 | 0.80 | . 00786 | 3.00 | . 0794 |
| 1.35 | . 126 | 0.85 | . 00873 | 3.10 | . 0841 |
| 1.40 | . 134 | 0.90 | . 00965 | 3.20 | . 0889 |
| 1.45 | . 143 | 0.95 | . 0106 | 3.30 | . 0938 |
| 1.50 | . 152 | 1.00 | . 0116 | 3.40 | . 0988 |
| 1.55 | . 161 | 1.05 | . 0126 | 3.50 | . 104 |
| 1.60 | . 170 | 1.10 | . 0137 | 3.60 | . 109 |
| 1.65 | . 180 | 1.15 | . 0148 | 3.70 | . 115 |
| 1.70 | . 189 | 1.20 | . 0160 | 3.80 | . 120 |
| 1.75 | . 199 | 1.25 | . 0172 | 4.00 | . 131 |
| 1.80 | . 209 | 1.30 | . 0184 | 4.20 | . 143 |
| 1.85 | . 219 | 1.35 | . 0196 | 4.40 | . 155 |
| 1.90 | . 230 | 1.40 | . 0209 | 4.60 | . 168 |
| 1.95 | . 240 | 1.45 | . 0222 | 4.80 | . 181 |
| 2.0 | . 251 | 1.50 | . 0236 | 5.00 | . 194 |
| 2.1 | . 274 | 1.55 | . 0250 | 5.20 | . 208 |
| 2.2 | . 297 | 1.60 | . 0264 | 5.40 | . 222 |
| 2.3 | . 321 | 1.65 | . 0279 | 5.60 | . 237 |
| 2.4 | . 346 | 1.70 | . 0294 | 5.80 | . 252 |
| 2.5 | . 371 | 1.75 | . 0309 | 6.00 | . 267 |
| 2.6 | . 398 | 1.80 | . 0324 | 6.20 | . 283 |
| 2.7 | . 425 | 1.85 | . 0341 | 6.40 | . 299 |

TABLE A1: FRICTION HEAD LOSSES
(continued)

| $\mathrm{d}=2.0^{\prime \prime}=.0557 \mathrm{~m}$ |  | $\mathrm{d}=2.0^{\prime \prime}=.0557 \mathrm{~m}$ |  | $\mathrm{d}=2.5^{\prime \prime}=.0674 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q, l/sec | $h_{\mathbf{f}} / \mathrm{L}$ | Q, 1/sec | $\mathbf{h f}_{\mathbf{f}} / \mathrm{L}$ | Q, l/sec | $h_{f} / \mathrm{L}$ |
| 0.2 | . 000236 | 2.90 | . 0255 | 0.40 | . 000322 |
| 025 | . 000300 | 3.00 | . 0270 | 0.50 | . 000475 |
| 0.30 | . 000481 | 3.20 | . 0303 | 0.60 . | . 000654 |
| 0.35 | . 000630 | 3.40 | . 0337 | 0.70 | . 000856 |
| 0.40 | . 000796 | 3.60 | . 0372 | 0.80 | . 00108 |
| 0.45 | . 000978 | 3.80 | . 0409 | 0.90 | . 00133 |
| 0.50 | . 00118 | 4.00 | . 0447 | 1.00 | . 00160 |
| 0.55 | . 00139 | 4.20 | . 0487 | 1.10 | . 00189 |
| 0.60 | . 00162 | 4.40 | . 0529 | 1.20 | . 00220 |
| 0.65 | . 00186 | 4.60 | . 0571 | 1.30 | . 00253 |
| 0.70 | . 00212 | 4.80 | . 0615 | 1.40 | . 00288 |
| 0.75 | . 00239 | 5.00 | . 0661 | 1.50 | . 00325 |
| 0.80 | . 00268 | 5.20 | . 0708 | 1.60 | . 00364 |
| 0.85 | . 00298 |  |  | 1.70 | . 00404 |
| 0.90 | . 00329 |  |  | 1.80 | . 00447 |
| . 0.95 | . 00361 |  |  | 1.90 | . 00492 |
| 1.00 | . 00395 |  |  | 2.00 | . 00538 |
| 1.10 | . 00467 |  |  | 2.10 | . 00586 |
| 1.20 | . 00544 |  |  | 2.20 | . 00635 |
| 1.30 | . 00626 |  |  | 2.30 | . 00687 |
| 1.40 | . 00712 |  |  | 2.40 | . 00740 |
| 1.50 | . 00804 |  |  | 2.50 | . 00794 |
| 1.60 | . 00900 |  |  | 2.60 | . 00851 |
| 1.70 | . 0100 |  |  | 2.70 | . 00909 |
| 1.80 | . 0111 |  |  | 2.80 | . 00969 |
| 1.90 | . 0122 |  |  | 2.90 | . 0103 |
| 2.00 | . 0133 |  |  | 3.00 | . 0109 |
| 2.10 | . 0145 |  |  | 3.20 | . 0122 |
| 2.20 | . 0157 |  |  | 3.40 | . 0136 |
| 2.30 | . 0170 |  |  | 3.60 | . 0150 |
| 2.40 | . 0183 |  |  | 3.80 | . 0165 |
| 2.50 | . 0196 |  |  | 4.00 | . 0180 |
| 2.60 | . 0210 |  |  | 4.20 | . 0197 |
| 2.70 | . 0225 |  |  | 4.40 | . 0214 |
| 2.80 | . 0240 |  |  | 4.60 | . 0231 |
|  |  |  |  | 4.80 | . 0249 |

TABLE A1: FRICTION HEAD LOSSES
(continued)

| $\mathrm{d}=2.5^{\prime \prime}=.0674 \mathrm{~m}$. |  | $\mathrm{d}=2.5{ }^{\prime \prime}=.0674 \mathrm{~m}$ |  | $\mathrm{d}=3.0^{\prime \prime}=.0820 \mathrm{~m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q, 1/sec | $h_{f} / \mathrm{L}$ | Q, 1/sec | $h_{f} / \mathrm{L}$ | Q, l/sec. | $h_{f} / \mathrm{L}$ |
| 5.0 | . 0267 | 9.8 | . 0868 | 0.50 | . 000187 |
| 5.2 | . 0286 | 10.0 | . 0899 | 0.75 | . 000380 |
| 5.4 | . 0305 | 10.5 | . 0979 | 1.00 | . 000628 |
| 5.6 | . 0326 | 11.0 | . 106 | 1.25 | . 000930 |
| 5.8 | . 0347 | 11.5 | . 115 | 1.50 | . 00128 |
| 6.0 | . 0368 | 12.0 | . 124 | 1.75 | . 00167 |
| 6.2 | . 0389 | 12.5 | . 133 | 2.00 | . 00211 |
| 6.4 | . 0412 | 13.0 | . 142 | 2.25 | . 00260 |
| 6.6 | . 0434 | 13.5 | . 152 | 2.50 | . 00312 |
| 6.8 | . 0458 | 14.0 | . 162 | 2.75 | . 00369 |
| 7.0 | . 0482 | 15.0 | . 183 | 3.00 | . 00430 |
| 7.2 | . 0506 | 16.0 | . 205 | 3.25 | . 00494 |
| 7.4. | . 0531 | 17.0 | . 228 | 3.50 | . 00563 |
| 7.6 | . 0556 | 18.0 | . 251 | 3.75 | . 00635 |
| 7.8 | . 0582 | 19.0 | . 276 | 4.00 | . 00711 |
| 8.0 | . 0608 | 20.0 | . 302 | 4.25 | . 00791 |
| 8.2 | . 0636 | 21.0 | . 329 | 4.50 | . 00874 |
| 8.4. | . 0662 | 22.0 | . 357 | 4.75 | . 00960 |
| 8.6 | . 0690 |  |  | 5.00 | . 0105 |
| 8.8 | . 0719 |  |  | 5.25 | . 0114 |
| 9.0 | . 0748 |  |  | 5.50 | . 0124 |
| 9.2 | . 0777 |  |  | 5.75 | . 0134 |
| 9.4 | . 0807 |  |  | 6.00 | . 0145 |
| 9.6 | . 0837 |  |  | 6.25 | . 0155 |

## TABLE A1: FRICTION HEAD LOSSES

(continued)

| $\mathbf{d}=\mathbf{3}^{\prime \prime}=. \mathbf{0 8 2 1 m}$ |  | $\mathbf{d}=\mathbf{3}^{\prime \prime}=. \mathbf{0 8 2 1 m}$ |  | $\mathbf{d = 3}=\mathbf{3}=\mathbf{0 8 2 1 m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{Q , \mathbf { l } / \mathbf { s e c }}$ | $\mathbf{h f} / \mathbf{L}$ | $\mathbf{Q , \mathbf { l } / \mathbf { s e c }}$ | $\mathbf{h} \mathbf{f} \mathbf{L}$ | $\mathbf{Q , \mathbf { l } / \mathbf { s e c }}$ | $\mathbf{h} \mathbf{f} / \mathbf{L}$ |
| 6.50 | .0166 | 12.00 | .0486 | 18.00 | .0988 |
| 6.75 | .0178 | 12.50 | .0522 | 18.50 | .103 |
| 7.00 | .0189 | 13.00 | 0559 | 19.00 | .109 |
| 7.50 | .0214 | 13.50 | .0597 | 20.00 | .119 |
| 8.00 | .0239 | 14.00 | .0637 | 21.00 | .129 |
| 8.50 | .0266 | 14.50 | .0677 | 22.00 | .140 |
| 9.00 | .0293 | 15.00 | .0718 | 23.00 | .152 |
| 9.50 | .0323 | 15.50 | .0761 | 24.00 | .164 |
| 10.00 | .0353 | 16.00 | .0804 | 25.00 | .176 |
| 10.50 | .0385 | 16.50 | .0849 | 26.00 | .188 |
| 11.00 | .0418 | 17.00 | .0894 | 27.00 | .201 |
| 11.50 | .0451 | 17.50 | .0941 | 28.00 | .214 |

Correction for slightly different pipe diameters: The friction head losses vary a lot with pipe diameter. For instance, with the 1" pipe, if you had used its nominal diameter, $1^{\prime \prime}=.0254 \mathrm{~m}$ instead of the S.D.R. 26 diameter of .0300 m , say, with $\mathrm{Q}=$ $0.8 \mathrm{l} / \mathrm{sec}$., equation A2 says that $\mathrm{h}_{\mathrm{f}} / \mathrm{L}$ would be .111 instead of .0506 (twice as much!). So, even pipes of the same nominal diameter but of different thickness have different friction head losses and once in a while you might want to correct for that. You can, of course, use the correct diameter in equation A2a. But if you don't have the right pocket calculator for that, you can correct Table A1 this way: Call $\mathrm{d}_{\mathrm{t}}$ the diameter of the pipe given in the table and $\mathrm{h}_{\mathrm{ft}}$, the corresponding head loss at a given flow rate. For a real diameter $d$ at the same flow rate, the corrected head loss is given by:

$$
\begin{equation*}
\mathrm{H}_{\mathrm{f}}=\left(\mathrm{h}_{\mathrm{f}}\right)_{\mathrm{t}} \times\left\{1+3.75\left(\mathrm{~d}_{\mathrm{t}}-\mathrm{d}\right) / \mathrm{d}_{\mathrm{t}}\right\} \tag{A5}
\end{equation*}
$$

Note that the smaller diameter pipe gives the larger head loss.

## A-II. The hydraulic grade line with a sock.

The HGL is modified $b$, the presence of a stationary sock (see figure A-2). As we have seen, the pressure does not change along the length of the sock either in the air or in the water. So, from the high point where the sock begins to its end, the HGL is parallel to the pipe profile standing vertically above it, a height equal to the value of h at the high point. At the end of the sock, where the pipe is filled with water again, the HGL resumes a slope whose sine is $h_{f} / L$.


Figure A-2

## A-III: Valves and pipe selection near high points

1) Enforcing subcritical flow near a valve

This matter is a little involved:
When one chooses to place an automatic air valve "at" a high point of a conduction line, one should take account of the following:
a) The valve should operate in a zone of subcritical flow in order to drain steadily slowly accumulating air and avoiding reacting to the rapid passages of bubbles carried by supercritical flows. The section of pipe into which the valve is embedded must, therefore, be large enough in diameter to impose subcritical flow at the highest possible flow rate, $\mathrm{Q}_{\text {max }}$. Practically, this means that the beginning of this section should be upstream of the valve and the end, some distance downstream- probably a couple of pipe lengths is enough.
b) The valve needs imperatively to be downstream of the high point (HP) because if it is upstream instead, the upstream extent of the sock stopping within one diameter or so of the HP, the valve will drain no air.
c) The steady formation of a stationary air pocket (in flowing water), "the sock", is only possible downstream of but starting at a sill ${ }^{2}$, meaning at a horizontal section of the pipe. If the subcritical section of pipe starts not at the HP but downstream of it, there will be no sill and the air pocket will be unstable and may miss the valve. So, it is necessary for the subcritical section to straddle the HP.
d) Finally, one must allow for an error in the determination of the location of the HP. For one thing, the HP for the pipe at the bottom of the trench may not be located where the HP of the surface of the trench used to be at survey time. How accurate does one have to be? The vertical accuracy is measured in units of pipe diameters (an error of one pipe diameter is a serious error). But near a HP, elevation differences are by definition small (of second order in the distance from it). So, one may misjudge the high point location by horizontal distances that may be appreciable, the more so, the flatter the ground around the HP.

So, one will opt for a conservative local design, one that will function well, even if a plausible error in the appreciation of the high point occurs. Let us call that error in the high point location $E_{U}$ if the HP is estimated upstream of where it really turns out to be and $E_{D}$ if the HP is estimated downstream of where it really turns out to be. Then, according to a) - d), the valve location should be at least $E_{U}$ downstream of the estimated HP whereas the beginning of the subcritical section should be at least ED upstream of the estimated HP. This means that one needs to:

- provide a subcritical section with a length of $\left(\mathrm{E}_{\mathrm{D}}+\mathrm{E}_{\mathrm{U}}+\mathrm{L}_{T}\right)$, where $\mathrm{L}_{T}$ is the length of trailing subcritical pipe downstream of the valve, say two pipe lengths.
- Place this segment so that the estimated HP is located $\mathrm{E}_{\mathrm{D}}$ downstream of the start of the subcritical section.

But what about $\mathrm{E}_{\mathrm{U}}$ and $\mathrm{E}_{\mathrm{D}}$ ? These possible errors will vary a great deal with the type of topography: small at a sharp ridge, large along a flat plateau. So, attention is needed to the topography near the high points:

- First, one should mark the surveyed high point with a stake.
- Second, several points should be surveyed on either side along the trench down to where the height of the ground is unambiguously less by a sufficient amount. Practically, a local slope of $+/-1^{0}$ seems adequate to define the end points of the lengths $\mathrm{E}_{\mathrm{D}}$ and $\mathrm{E}_{\mathrm{U}}$ respectively. Provided the topography is sufficiently detailed, the evaluation of the length of subcritical pipe required for a given case and of the location of the airrelease valve are conveniently provided by the ALV Air-In-Pipes Visual Basic program.

[^2]
## Enforcing a supercritical flow downstream of a high point

If a supercritical flow is desired downstream of a high point to prevent the presence of stationary air socks, fewer precautions need to be taken. Since no automatic valve is involved, it does not make much difference whether the supercritical section occurs immediately upstream or downstream of the high point. In the later case, only an unimportant air pocket of negligible vertical extent will result. On the other hand, the downstream end of the supercritical pipe section should not coincide with the end of the otherwise possible air sock but instead, it should extend to the low point following the high point in question. The reason is given in Appendix B-III.

## A-IV: How to combine pipe diameters to get a given

## friction head loss over a given length

and with a given flow rate.
Let the required head loss be $h_{a}$ and let the length be L. Divide one by the other to get $h_{a} / L$. Let us assume that you have already selected one of the pipe diameters, but that it is too small to be used over the whole length $L$ of the pipe. Call this diameter $\mathrm{d}_{\mathrm{S}}$ and the corresponding friction loss per unit length, $\left(\mathrm{h}_{\mathrm{f}} / \mathrm{L}\right)_{\mathrm{S}}$. For the required value of Q find in table A 1 a diameter $\mathrm{d}_{1}$ which causes a $\mathrm{h}_{\mathrm{f}} / \mathrm{L}$ smaller than $h_{a} / L$. Call this value from the table $\left(h_{f} / L\right)_{1}$. The two pipe lengths which together will add up to $L$ and cause a friction head loss equal to $h_{a}$ are given by:

$$
\begin{gather*}
\mathrm{L}_{\mathrm{l}}=\left\{\mathrm{L}\left(\mathrm{~h}_{\mathrm{f}} / \mathrm{L}\right)_{\mathrm{S}}-\mathrm{h}_{\mathrm{a}}\right\} /\left\{\left(\mathrm{h}_{\mathrm{f}} / \mathrm{L}\right)_{\mathrm{S}}-\left(\mathrm{h}_{\mathrm{f}} / \mathrm{L}\right)_{\mathrm{l}}\right\}  \tag{A4}\\
\mathrm{L}_{\mathrm{s}}=\mathrm{L}-\mathrm{L}_{1}
\end{gather*}
$$

where $L_{l}$ is the length of the pipe of larger diameter and smaller friction and $L_{S}$ is the length of the pipe with smaller diameter and larger friction.

Note: The larger pipe diameter does not have to be the size just above the smaller one. For instance, if $d_{S}=3 / 4^{\prime \prime}, \mathrm{d}_{1}$ can be 1.5 " instead of 1 ".

## A -V. Table of Critical flow rates

These are defined in Chapter I. Recall that a flow rate larger than $\mathrm{Q}_{s}$ is supercritical and will sweep air bubbles along with it while a flow rate smaller than $Q_{c}$ is subcritical and will allow stationary air pockets downstream of high points. The table makes use of Equations (2a) and (2b). But here, while the pipe diameters are in meters, the flow rates are in liters /second.

| d, nominal | $\mathrm{d},(\mathrm{ID}, \mathrm{m})$ | Qc (I/sec) | Qs (I/sec) |
| :---: | :---: | :---: | :---: |
| $1 / 2^{"}$ | 0.0173 | 0.047 | 0.0618 |
| $3 / 4 "$ | 0.0231 | 0.097 | 0.1273 |
| $1 "$ | 0.03 | 0.186 | 0.2447 |
| $1.25 "$ | 0.0389 | 0.355 | 0.4686 |
| $1.5^{\prime \prime}$ | 0.0444 | 0.494 | 0.6522 |
| $2 "$ | 0.0557 | 0.871 | 1.1496 |
| $2.5^{\prime \prime}$ | 0.0674 | 1.403 | 1.8516 |
| $3^{\prime \prime}$ | 0.083 | 2.362 | 3.1160 |

## APPENDIX B

B-I: Why is the volume of a sock inversely proportional to its absolute pressure? This does not happen right away. As the pressure increases in the sock, the volume decreases but, to start with, less than the formula predicts, because the air temperature increases (the compression is initially close to isentropic). After a while, the air in the pipe cools off by losing heat to the pipe and to the dirt in the trench so that its temperature is eventually the same as before the compression. Only then does the volume of air follow the law given in Appendix A-II. So, for those cases A1 which are close to cases B, you may have to wait a while before water comes out at the end of the pipe.

## B-II: Can't you get rid of the air in a sock permanently with a small hole drilled in the cap of a $\mathbf{T}$ ?

In the absence of an automatic air-flushing valve, it is tempting to drill a small hole in a plastic T cap, for instance, because such a hole will let a lot more air escape than water. This solution would surely simplify things. The trouble with it is that the hole has to be rather small not to waste an excessive amount of water in a typical installation. So, it has to be made carefully and it is likely to plug up. The relation between the hole diameter d , the flow rate Q of water through the hole and the head $\mathrm{h}_{1}$ in the pipe at the hole is approximately, (this formula is not too accurate for small holes)

$$
\mathrm{d}=0.3 \mathrm{Q}^{1 / 2 / \mathrm{h}_{1}} 1 / 4
$$

where the hole diameter is in meters, the flow rate is in $\mathrm{m}^{3} / \mathrm{sec}$. and the head is in meters. The same size hole will pass a volume of air about 28 times as large. It does not make much difference whether the hole is at the top or at the bottom of the pipe.
Now if, for example, we want no more than $3 \%$ of the water flow rate through the pipe to spill out of it, if the pipe passes $15 \mathrm{lit} . / \mathrm{min}$. of water and if the head at the sock is 10 meters, we get for the diameter of the hole:

$$
\mathrm{d}=0.3 \times\left\{15 \times .03 / 60 \times 10^{3}\right\}^{1 / 2} \times 10^{-1 / 4}=0.00046 \mathrm{~m}=0.46 \mathrm{~mm}
$$

A hole diameter twice as big will waste 4 times as much water.
So, this solution may be handy at times but I would worry about dirt or vegetable matter plugging it up. Don't expect too much luck in general with small holes passing a reliable amount of water or not plugging up completely.

## B-III: More on changing the diameter in the sock area.

a) When the design calls for a smaller diameter pipe along the sock than elsewhere, this manual recommends that the smaller pipe extends to the low point rather than only to the end of the sock. Why?

As Q increases and approaches $\mathrm{Q}_{\mathrm{C}}$, the top of the sock moves along the downhill leg downstream of the high point. But it does not get chased out of the downhill leg suddenly. This is because, for moderate pipe angles, the flow rate required to chase the pocket downstream first increases (from the horizontal to about 15 degrees of slope) then decreases. You need about the same flow rate to chase the top of the sock for a horizontal pipe and for one that has about 35 degrees of slope. As a result, if the pipe diameter is increased between C and $\mathrm{C}^{\prime}$ for a flow which is supercritical with respect to the smaller diameter, but subcritical with respect to the larger one, you should expect the sock to be trapped at the section where the diameter changes unless that section is at the bottom of the pipe.
b) Start up: Note that to choose a smaller diameter at the lower end of sections, such as $B C^{\prime}$, than at the upper end can help with the starting problem because the compression head $h_{1}$ will then shorten the initial sock more than with a uniform diameter ( $1 / / l^{\prime}$ will then be smaller than $\mathrm{v} / \mathrm{v}^{\prime}$ in the calculation of the trickle height). But you also have to take into account the effect of this change of diameter on the steady operation of the system after start up so that this trick can seldom be used. Anyway, you don't really need it.

## B-IV: How do I control the water velocity in the conduction line?

The advice commonly given is to keep the water velocity between limits, such as $0.7 \mathrm{~m} / \mathrm{sec}$ and $3.0 \mathrm{~m} / \mathrm{sec}$. The reason is that if the velocity is too low, sediments will tend to deposit in the pipe, especially at low points and eventually plug up the conduit, while if the velocity is too high, the same sediments will tend to erode the pipes. Lateral forces at elbows may also require special immobilizing measures when the water velocity is high.

It is usually easy to keep water from exceeding the recommended upper limit. However, it's not so to keep it above the lower limit because it might require abandoning other more crucial constraints. For instance, your system may not have enough slope to allow that velocity.
(Note from Table A-3 that all subcritical velocities for pipes with diameters up to 3 " fall below the recommended lower limit). This situation is a bit like the one we face with the Ten Commandments: We do our best but sometimes we will sin. Then, we make amends. For instance, if a short section of pipe passes under a stream (a favorite spot for the accumulation of sediments), you can usually afford the extra friction loss caused by a pipe of small enough diameter to keep the velocity high along that short passage. Now, if the pipe then climbs for a considerable length towards the tank, you may need further "amends": you will provide a clean-out near the river crossing and you will make sure that either as it enters the spring box, or within a sedimentation unit next to it, the spring water is forced to filter most of its sediments.

## B-V: Can't one operate with a full pipe all the time by adding a regulating valve at the end of the conduction line in order to match its head loss to that required for the variable flow rate of the spring?

This sounds great and it would make it unnecessary to read most of this complicated manual! But if you try it, you will find that it takes days and repeated air bleedings to reach the proper valve setting. So, the villager will either not manage trouble free operations (valve opened too much) or close it down too much and deprive himself of water that could be delivered.

## B-VI: Where does equation (1) come from?

This is an experimental result of sufficient accuracy for practical use. It also has some theoretical support. Its origin may only interest hydraulics specialists. Nevertheless (since it is the basis of this manual), it is given below:
A) This equation applies evidently to air pockets that are long compared to the diameter of the pipe, and not to bubbles, such as those found in water levels. For the later $Q_{c}=0$ but $h_{t}$ is negligible and so, they are unimportant.
B) To be perfectly general, we should write (from Dimensional Analysis),

$$
\mathrm{Q}_{\mathrm{c}}=\mathrm{Ad}^{5 / 2} \mathrm{~g}^{1 / 2}
$$

Where the numerical value of A depends on:

- the surface tension of water (the Weber number, $16 \mathrm{Q}^{2} / \pi^{2} \mathrm{~d}^{3} \gamma$, where $\gamma$ is the surface tension as well as on the contact angle.
- the velocity profile, through the Reynolds number, $4 Q / \pi d v$, where $v$ is the kinematic viscosity of water, - the pipe slope distribution in the region of the pocket.

Now:

1) For a non-viscous fluid flow with negligible surface tension and a horizontal pipe, it is possible to calculate theoretically the velocity of propagation of the nose of a semi infinite ${ }^{3}$ air bubble into a water filled circular pipe, (the water ahead of the air pocket being at rest). This gives, (see Brooks Benjamin, Journal of Fluid Mechanics, 1968, vol.31, pages 209-248):

$$
\mathrm{Q}_{\mathrm{c}}=0.426 \mathrm{~d}^{5 / 2} \mathrm{~g}^{1 / 2}
$$

2) The real case is different because:

- It is the water that flows while the bubble does not move. (This is a trivial difference).

[^3]- Water is a viscous fluid and its velocity far upstream of the sock is not uniform across the pipe. It is zero everywhere at the pipe wall.
- There is some (though small) surface tension between the water and the air along the bubble-water boundary near the bubble nose.
- When you want to chase the bubble not only from the high points but all the way past the next low points so that the head of the bubble will move past inclined sections of the pipe, the simple theory for the horizontal pipe fails to yield conclusive information about the existence of a bubble propagation velocity.

3) Experimentally, but still for a horizontal section of pipe with typical Weber and Reynolds numbers and with the water flowing, the critical flow rate (the flow rate for which a long bubble remains stationary) is about:

$$
\mathrm{Q}_{\mathrm{c}}=0.38 \mathrm{~d}^{5 / 2 /} \mathrm{g}^{1 / 2}
$$

The bubble moves upstream if Q is less (but never if the pipe upstream bends down) and it moves downstream if Q is more (but not necessarily more than a short distance if the pipe downstream bends down).
4) Finally, also experimentally, if a section of straight pipe is inclined (lowered downstream) through variable angles, $Q_{c}$ first increases as the slope increases from the horizontal to a maximum of about 35 degrees and then decreases as the slope continues to increase past that value. The value of the constant $A$ chosen for $Q_{s}$, i.e 0.5 is slightly larger than that which is needed in equation (1) to flush the sock of the worst slope ( 30 degrees) somewhere along the sock. If the maximum slope downstream of a high point is small (say, 5 or 10 degrees) the value of A chosen is somewhat too large.

The experimental results referred to above are all mine.

B-VII: Is it possible in the field to choose a conduction line that minimizes air problems?

It is not only possible but often easy and always highly recommended.
One attempts the four following steps:

1) Avoid high points. Many will be unavoidable. At least beware of the unintentional ones (see example problem 5).
2) Between a high point high along the conduction line (towards the spring) and one lower down, choose the lower one. This will result in a higher pressure and therefore a lower volume within the sock.

3) Between two equally high high points followed by low point of unequal heights, choose the one with the higher low point.

4) Finally- and this one is frequently an effective option, for two high points of equal height followed by two low points of identical height, choose the one for which the trench profile maintains its height longest and drops down to the low point latest. The effect of this simple option is especially large when the pressure at the high point is substantially larger than atmospheric.


[^0]:    * See for instance: A Handbook of Gravity-Flow Water Systems by Thomas D. Jordan, Intermediate Technology Publications, 1984. This handbook covers a comprehensive list of topics including "air blocks". While incomplete for our purposes, Jordan's discussion of air in pipes is nevertheless a valuable start.

[^1]:    ${ }^{1}$ Blasius's formula applies only to smooth pipes, such as PVC. It also depends on the viscosity of the fluid. Here, the kinematic viscosity of the water has been assumed: $v=9.8 \mathrm{E}^{-7} \mathrm{~m}^{2} / \mathrm{sec}$, which corresponds to a water temperature of about 21 degrees centigrade. This formula is accurate enough up to a value of $\mathrm{Q} / \mathrm{d}=$ approximately 100 (liters/sec)/meter which is the useful range of the tables.

[^2]:    ${ }^{2}$ for the same reason that in free surface flow in canals, (as shown in long wave theory) the steady transition from subcritical to supercritical flow occurs only over a sill.

[^3]:    ${ }^{3}$ Because the flow some distance downstream of the nose of the bubble becomes supercritical in the sense: Froude Number>1, downstream signals do not propagate upstream so that the bubble does not really have to be semi-infinite, only long in the sense of shallow wave theory.

