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Notes on Suspension Pipe Bridges (3/1998)

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These notes assume no previous background and provide all relevant derivations. They cover suspension of horizontal and slanted beds. But the study is restricted to bridges for which the towers or pillars have equal height above their respective beds.
For use by field workers these notes should be replaced by simpler ones in Spanish giving only the principal results.
These notes as well as the simpler ones are complemented by an Excel program called ABRIDGE,(due to G. Corcos and J.P. Vial) which automates the design of such bridges by incorporating the equations and construction procedures discussed here.

Nomenclature:
$T(x)$ = Tension of the main cable inboard of pillars
α= angle of the straight line joining the bases of the pillars to the horizontal.
$\theta(x)$= angle between the cable (inboard of the pillars) and the horizontal
D= nominal diameter of main cable, (m).
d= internal diameter of bridge pipe (m).
L=bridge span= horizontal spacing between the two (inboard faces of) pillars.
E= Vertical sag of the cable (measured at the mid-span with respect to the straight line between the points of attachment of the cable on the pillars.
Z_1= Lateral displacement of anchor attachment point from pillar top when two anchors are used for each pillar.
w= bridge weight /unit horizontal length. (in what follows w is assumed independent of x).
W= total bridge weight.
W_1= Weight of the part of the bridge included with the main cable at the time the cable is first secured to the two pillars.
x= horizontal coordinate in a vertical plane containing the pipe. Its origin is the point where the tangent to the cable is parallel to the tangent to the pipe.. When the bridge bed (pipe)is slanted, x is >o on the high side of the bed.

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x_c= linear coordinate along the cable with origin at point where the tangent to the cable is parallel to the tangent to the pipe.
y=vertical coordinate associated with x.
z,= coordinate perpendicular to x and y. $_n$
h_n = hanger length for nth hanger, = distance between the pipe and de main cable along the hanger.
H= height of the cable attaching points on top of the columns with respect to the ground. (m)
h_t=additional column height above attaching point of cables
m=net length of center hanger, (m)
CON= convexity of piping (m)
e= height of pipe ends above ground
l_{nc}= length along the cable of the location of the nth hanger.
The origin of coordinates is taken to be the midpoint of the straight line parallel to the bridge bed, (the pipe) but touching the cable there.

<u>Derivation:</u>

For a cable in a vertical plane assumed deprived of stiffness, (i.e. all loads are axial) and complemented by a sufficient ("almost infinite") number of vertical (also slender) hangers transmitting loads :

$$\sum F_x = 0 \qquad \frac{d}{dx}(T\cos\theta) = 0 \text{ or } T\cos\theta = T_x = T_0 = \text{constant}$$

This follows from the fact that the hangers are vertical and also only capable of handling axial loads.

$$\sum F_y = 0 \qquad \frac{\mathrm{d}}{\mathrm{dx}}(T\sin\vartheta) = w$$

In the second equation for T substitute $T_0/\cos\theta$ and note that tan θ= dy/dx along the cable. Hence the two equations yield together

$$d^2y/dx^2 = w/T_0; \qquad (3)$$

Integrating, we get

$$y = wx^2/2T_0 + Ax + B; \quad (4)$$

There are three undetermined constants: A, B and T_0.

a) At $x= -L/2$, $y= wL^2/8T_0-AL/2+B= E+ h_i$ where h_i is $y(-L/2)$, at the "left-hand pillar" = $-(L/2)\tan\alpha$.

b)At $x=+L/2$, $y = wL^2/8T_0+AL/2 +B=E+h_d$ where h_d is $y(+L/2)$ at the "right hand pillar" $=+(L/2)\tan\alpha$

Subtracting a) from b) we find that $A= \tan \alpha$, while adding them we find that

$$B=E-wL^2/8T_0$$

So we write the cable equation

$$y= w (x^2-L^2/4)/2T_0+ x \tan\alpha+E$$

Next we determine T_0. At the as yet undetermined point of tangency x_t between pipe and cable, $y=y_t$. both the coordinates and the slopes of the pipe and the cable correspond. So at the point of tangency, first equating the heights for cable and straight pipe bed:

$$\frac{w}{2T_0}\left(x_t^2-\frac{L^2}{4}\right)+x_t\tan\alpha+E=x_t\tan\alpha \quad ; \qquad (5)$$

or $$\frac{w}{2T_0}\left(\frac{L^2}{4}-x_t^2\right)=E: \qquad (6)$$

and equating slopes:

$$\frac{w}{T_0}x_t+\tan\alpha=\tan\alpha$$

So that $x_t=0$: the point of tangency is at the origin, half way between the pillars even when the pipe bed has a slope. Now from (6) we see that

$$T_0=wL^2/8E=WL/8E; \qquad (7)$$

regardless of the slope. Notice also that the parabolic equation for the cable height has now become:

$$y=4E(x^2/L^2)+x \tan\alpha ; \qquad (8)$$

The lowest point on the cable, where the slope $dy/dx=0$ is obtained from

$$dy/dx=8Ex/L^2+\tan \alpha=0$$

$$\text{or}\quad x_0=-(L^2/8E)\tan\alpha$$

This point does not turn out to be particularly close to the mid point. In fact it will be found beyond one of the pillars if E is sufficiently small and α is sufficiently large.

The y coordinate of this low point is obtained by substituting the value of x_0 in the equation for cable y. One gets:

$$y_0=-(L^2/16E)\tan^2\alpha$$

The maximum tension T_{max} is given from

$$T_{max}=T_0/\cos\theta_{max}=T_0(1+\tan^2\theta_{max})^{1/2}$$

But tan θ =dy/dx so that:

$$T_{max} = T_0\sqrt{\left[1+\left\{\frac{dy}{dx}\right\}^2_{max}\right]}$$

Now

$$dy/dx=8Ex/L^2+\tan\alpha$$

so that

$$(dy/dx)^2= 64\,E^2x^2\,/L^4+ 16(Ex/L^2)\tan\alpha+\tan^2\alpha$$

and the maximum value is found at either L/2 or -L/2, (at the highest pillar) so that

$$T_{max} = T_0\sqrt{\left[1+\left(\frac{4E}{L}+|\tan\alpha|\right)^2\right]}$$

and since from equ.(7), T_0=WL/8E,

$$T_{max} = \frac{WL}{8E}\sqrt{1+\left(\frac{4E}{L}+|\tan\alpha|\right)^2} \qquad (9)$$

which for the symmetric case becomes

$$\frac{T_{\max}}{W} = \frac{L}{8E}\sqrt{1+\left(\frac{4E}{L}\right)^2} \qquad (10)$$

The inverse of equation (9) is

$$\frac{E}{L} = \frac{1+\tan^2\alpha}{\left[64\frac{T^2}{W^2}(1+\tan^2\alpha)-16\right]^{\frac{1}{2}} - 4\left|\tan\alpha\right|} \qquad (9a)$$

Note that with same total weight and horizontal span, T_{max} increases with α because T_0 is the same. Note also that T_{max} occurs just inboard of the pillar on the high side. But the change of T_{max} with α is small in general because α is rarely a very large angle.

In all the preceding and the following the symmetric case with a horizontal bed can be obtained by simply setting α =0 in the formulae.

The choice of E

It is clear from the above that to each E corresponds a T_{max} and that since T_{max} needs be inferior to the working tension for the cable there will be a minimum cable diameter associated with the chosen value of E. It turns out from equation (9) that very small values of E lead to very large cable tensions which are undesirable both for the reason that the construction of the bridge and of its columns becomes difficult and that the cable diameter becomes unwieldy and expensive, while large values of E lead rapidly to a tension that (for moderate values of the bed slope) approaches half the bridge weight so that high values of E (high columns) do not decrease the tension much and so are not desirable either. The practical range of E/L under usual rural conditions can be chosen as $0.03 < E/L < 0.07$, ($4.2 > T/W > 1.85$, if $\alpha=0$). For large road bridges on the other hand a typical value of E/L is seldom lower than 0.11.

Three reasons why the sag may be smaller than the height of the columns above the pipe bed.

First where the pipe bed is tangent to the cable slope, it may not touch the cable but be suspended to the cable by a hanger.

Second it is esthetically pleasant, (I think) for the final pipe bed to have an upward convex shape. This can be achieved if the length of the hangers is chosen appropriately. Then the column height will be enhanced by that upward deflection of the pipe at its middle

Finally the pipe level may be raised above the base of the columns for various reasons such as the necessity for additional clearance between the pipe bed and the maximum level of of a river.

<u>The effect of the elastic stretching of the cable under load</u>
As we shall see shortly any stretch of the cable under an additional load results in an additional sag larger than the corresponding cable stretch so that to obtain a given sag E under a final load (e.g. cable weight+ hangers weight+ pipe weight+ water weight) one needs to select an initially smaller sag E_i corresponding to the initial load W_i supported by the cable when it is first fixed unto the two columns.(in practice these are always different).

Warning! The formulation of the relation between change of cable load and change of cable sag is given here. But the actual calculations required depend on the way the bridge is assembled. See later and in the Excel program ABRIDGE.

<u>Length of the Main Cable Between the Supports</u>

The length of the curved cable is given from:

$$L_c = \int_{-L/2}^{L/2} \sqrt{1+\left({dy}/{dx}\right)^2}\,dx$$

$$\approx \int_{-L/2}^{L/2} \left(1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2\right)dx$$

$$\text{and since dy/dx} = 8\text{Ex/L}^2 + \tan\alpha$$

$$L_c \approx L\left(1+ {}^{1}\!/_{2}\tan^2\alpha\right) + 8/3\,E^2/L$$

The exact integral is somewhat more cumbersome and not different enough to bother as long as $(dy/dx)^2$ is small compared to unity everywhere in the interval (-L/2 , L/2). The % error is bounded by $(dy/dx)^4_{max}$, e.g. for α=0 and E/L=0.1, the exact expression gives L_c= 1.0261L, the approximate one, L_c= 1.0267L. The difference grows however with the slope of the piping.

Note that

$$\partial E \big/ \partial L_c = (3/16)\,\frac{L}{E}, \quad \text{i.e. rather large}$$

so that the difference between the lengths of a cable with two different sags E_2 & E_1 is

$$L_{c2} - L_{c1} = 8\big/3\,\frac{E_2^2 - E_1^2}{L}$$

which implies that a difference in cable length is translated into a larger difference in sag. This warns us to take seriously the small % elongation in the cable due to its longitudinal stress. This elongation is made up of two parts: The first one is inelastic and has to do with the construction of the cable. Some of that one can be gotten rid of by pre-stretching the cable. Some, unfortunately, will remain thru repeated loading variations. The second is an elastic stress. In view of the fact that the stress and therefore the strain vary along the cable length (with the cable angle to the horizontal), we write Hook's Law as:

$$\frac{\partial\delta}{\partial x} = \frac{T(x)}{AE_m}\frac{\partial x_c}{\partial x}$$

where δ is the total elongation of the cable under tension, T is the local cable tension, x_c, the coordinate along the cable, A its nominal cross section, ($\pi D^2/4$, where D is the nominal cable diameter) and E_m the cable effective modulus of elasticity, which we will approximate for a steel cable by $8.3E^{10}$ Newtons/m.^{2} (or $8.44E^9$ kg/m^2, or $12E^6$ #/inch^2) as recommended in design manuals, (e.g. Shigley & Mitchell, Mechanical Engineering Design, McGraw-Hill, p.782).

Warning! The exact value of E_m is likely to vary with the cable steel and cable design so that efforts might be made to acquire it from the manufacturer, in view of the fact that it is quite impracticable to determine it in the field. However large variations in E_m occur only with special steel wire ropes which are not likely to be used in developing countries- a good thing because manufacturers are usually mute about the modulus of their cables.

Since:

$$T(x) = T_0\sqrt{1+\left(\frac{dy}{dx}\right)^2} \quad \text{and} \quad \frac{\partial\delta}{\partial x} = \frac{T(x)}{AE_m}\,{\partial x_c}\big/{\partial x} = \frac{T(x)}{AE_m}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{\partial\delta}{\partial x} = \frac{T_0}{AE_m}\left(1+\left(\frac{dy}{dx}\right)^2\right)$$

Now while

$$(dy/dx)^2 = \tan^2\alpha + \left(\frac{16Ex}{L^2}\right)\tan\alpha + \frac{64E^2x^2}{L^4}$$

only the even terms in x will contribute to the integration between symmetric limits

we get after performing the integration

$$\delta = \frac{T_0 L}{AE_m}\left[1+\tan^2\alpha + \frac{16}{3}\frac{E^2}{L^2}\right],$$

$$= \frac{WL^2}{8AEE_m}\left[1+\tan^2\alpha + \frac{16}{3}\frac{E^2}{L^2}\right]$$

where W is the load on the cable and δ is the stretch.

To every load W_1, W_2 , …W_n, corresponds a stretch δ_1, δ_2 ... δ_n. Note that the additional stretch caused by the difference between the cable loadings W_2 and W_1 is:

$$\delta_2 - \delta_1 = (8/3L)(E^2{}_2 - E^2{}_1)$$

Equating these two expressions for the additional stretch, we get

$$\delta_2 - \delta_1 = \frac{8}{3}\frac{E_2^2 - E_1^2}{L} =$$

$$= \frac{W_2 L^2}{8E_2E_m}\left(1+\tan^2\alpha + \frac{16E_2^2}{3L^2}\right) - \frac{W_1 L^2}{8E_1E_m}\left(1+\tan^2\alpha + \frac{16E_1^2}{3L^2}\right)$$

Or, if we want to determine the original sag E_i, (corresponding to a load W_i) required in order to obtain for a larger load W a desired sag E, we arrange the above equation as a polynomial equation in decreasing powers of E_i :

$$\frac{8E}{3L}E_i^{\,3}-\frac{2W_iE}{3AE_m}E_i^2+\left(\frac{WL^2}{8AE_m}\left\{1+\tan^2\alpha+\frac{16E^2}{3L^2}\right\}-\frac{8E^3}{3L}\right)Ei$$
$$-\frac{W_iL^2E}{8AE_m}\;1+\tan^2\alpha\;\;=0 \qquad (11)$$

The solution for E_i is a real and positive root of this third order equation. Instead of using standard mathematical techniques for extracting it, it is best obtained through Excel, using from the tool bar **tools** and from its menu, **Target value,** defining as target cell the one that contains the polynomial, its target value as **0** and the cell value to be modified as the one associated with E_i. In that cell, introduce an arbitrary initial value : (to avoid spurious roots it is best to use a value near that of E). ABRIDGE, the APLV automatic bridge design program incorporates this method.

NOTE: As mentioned above,the length of the main cable in between the two columns will vary as the load on the bridge varies. For the calculation of hanger lengths it is the final (maximum) weight of the bridge that is relevant hence in the calculation of the cable length it is the final sag that needs to be used in the equation of Page 6, while for the measure of the cable length to be initially installed and fixed to the two columns it is:

$$L_{ci}=L(1+1/2\tan^2\alpha)+8/3E_i^{\,2}/L^2$$

Practically though, it is best to set the initial sag E_i directly with a theodolite than by measuring the length of the cable between attachment points.

Alternate construction techniques.

In the field one normally erects a bridge without the help of a crane, or any device which can’t be hand-carried. Many methods have been improvised. We shall refer only to two of them.

- For short spans one can often first position the cable and then raise the piping to it.

- One method of construction that we have used successfully for bridges of large spans is to make use of a temporary auxiliary cable equipped with short loops all along its length. Through the loops are threaded the main cable its hangers and the piping. The unit is strung along first perpendicular to the bridge. The main cable(s) and the auxiliary cable are secured to the column on one side and the assembly is dragged or floated across the river or gap to the other side. The auxiliary cable is then pulled hard enough over the second column to allow the main cable to be fixed with (more or less) the chosen sag E_1. The auxiliary cable is then pulled out on the first side and the bridge falls in place. Care has to be exercised to avoid bending the piping excessively during the operation. Also when the auxiliary cable is pulled over column 2 it will, through friction, exert a strong pull outward onto that column and in that case it is best to secure it with a temporary stay or anchor inward.

Note that the previous considerations (the calculation of the initial sag required to obtain a desired final sag) apply for both of these construction techniques.

- In the first case the initial weight W_i is that of the cable + hangers and the final weight, W is the total weight of the bridge + water.

- In the second case, the auxiliary cable needs to be pulled enough for the sag of the supported main cable to be approximately the desired sag E_1. Approximately then, the sag of the auxiliary cable should be that of the main cable E_1 minus the length of the auxiliary cable loops. Approximately only because once the main cable has been secured to the second column and the auxiliary cable pulled out the main cable sag can be adjusted further, for instance if it is secured to a turnbuckle. Note that this cable sag is not the final sag E, since during the mounting of the bridge the pipe is empty of water.

These considerations show that you must on the site of the bridge be equipped with a level or for a slanted bridge with a theodolite to measure precisely intermediate and final sags.

The final completely loaded bridge needs to have a bed which is either plane or somewhat curved upward.

Examples: <u>Construction method a)</u>

With L=64.3m, W_p= The weight of the bridge – that of the cable=378 kg. E_2=2.25m,. Weight of cable + hangers = 23Kg , total weight =401kgs. $\alpha=8.66^0$, single cable. From the table <Tension> the cable diameter to chose is 7/16 " or .0111m.
When the cable is initially stretched it weighs only 23kgs. From the second Excel sheet, <Initial sag> we then get E_1=1.58m.

- <u>Construction method b,</u>

Using the same example 1: With L=64.3m, W_p= The weight of the bridge – that of the water=264kg. We use an auxiliary cable of 1/2inch , with weight 0,592 X 65 = 39 kgs so the whole assembly weighs 303kgs. After the auxiliary cable has been disposed of the main cable will need to have a sag E_1 with the pipe empty of water that corresponds to the weight 264 kgs while when full it is required to have a sag =2.25 meters with a weight of 401kgs.So E_1 is now sought first (sheet 2, <initial sag>: The calculation yields 2.07m. This in turn gives the designer an idea of the sag required of the auxiliary cable: If the loops along the auxiliary cable hang down, let us say, 30cms a value of the auxiliary cable sag of 2.07-0.30= 1.77m would (roughly) allow the sag of the main cable to be 2.07, assuming that this main cable is secured directly while the auxiliary cable is still holding it. It is recommended for the designer to have a means of further adjustment of the cable sag, for instance with a turnbuckle.

Note that the tension required of the auxiliary cable is according to the <tension > sheet or to equation (9) with 303 kgs and 1.77m of sag: 1,422kgs.

The height H of the columns above the pipe bed is not necessarily E:

The height of the columns or support H above the pipe bed is only equal to E if a) the main cable is tied without hanger at the tangent point between pipe bridge and main cable ; b) the final loaded pipe bed is to be straight ; c) the pipe bed is not elevated above the base of the two columns. But a) if for some reason there is a hanger at the closest separation between pipe and cable its length needs to be added to obtain the columns height. And b) if we wish the pipeline to assume an upward convex shape, (which I find esthetically desirable) the height of the supporting columns will have to be correspondingly larger and the shape of the pipeline will be determined by a judicious (though very simple) modification of the hanger lengths.

There are various possible contributions to the height of the columns above ground.

Hanger lengths.

It is worth mentioning that the purpose of suspension bridges is to minimize the bending loads on the pipes. If as is the case with our pipe bridges, the load is uniform with x, the bending load on the pipes will be minimized provided the cable shape is parabolic. But it is the length distribution of the hangers which will confer this shape on the cable (not the uniformity of the bridge load, as is often asserted in elementary textbooks). Hence it is important to control accurately the lengths of the hangers.

If the pipe is straight when the bridge is loaded and the cable is tied directly to the pipe at their tangent point the net distance between the top of the pipe and the main cable is given by

$$Y_c\text{-}Y_p=4E(x/L)^2\,; \qquad (13a)$$

In this case and if the pipe bed is not raised with respect to the base of the columns, E is both H, the height of the cable attaching point on the pillars above the pipe bed and the sag of the cable.
If the cable is tied to the piping at their tangent point thru a hanger of length l,

$$Y_c\text{-}Y_p= 4E(x/L)^2+ l \qquad (13b)$$

And the pillars height is H=E +l

And if in addition one chooses a slightly inverted parabola for the ultimate shape of the pipeline with, say an inverted pipe sag of value C_{on} (positive), the hangers lengths are,

$$Y_c\text{-}Y_p= 4(E+C_{on})(x/L)^2 \ +l \qquad (13c)$$

in this case the pillars have height $H=(E+C_2+l)$ if the desired ultimate sag is E.
Note that x is not the length of pipe from the center measured along the pipe but its horizontal coordinate. The corresponding distance along the pipe is x/cosα

It is useful to introduce into (13a), (13b) or (13c) the standard uniform spacing chosen for the hangers. For instance with 6m galvanized iron pipes, allowing for 3cms of gap between the pipes at the unions, and assuming two hangers per pipe, we will have a sequence n of hangers ranging from n=0, (at the origin of x and x_c) to the largest absolute value of n smaller than L/6.03m and, designating the hanger length corresponding to n by h_n,

$$h_n= 36.36\ (E+Con)(n\cos\alpha/L)^2 \ \ +l$$

Substitute $E+C_{on}$ for E when the pipeline curves upwards (negative pipe sag=C_{on}) is not rigorously accurate but sufficient.

Naturally to these lengths need to be added the length of cable necessary for fastening to the pipe and around the cable, perhaps 60 cms for each hanger.
<u>Locating the hangers along the cable.</u>

Returning to page 4, the length of cable from the attaching point x=0 to a point x is given by:

$$
\begin{aligned}
lc_x &= \int_0^x \sqrt{\left(1+\left[\frac{dy}{dx}\right]^2\right)}dx \\
&\approx \int_0^x \left[1+\frac{1}{2}\left(4Ex/L^2+\tan\alpha\right)^2\right]dx; \\
&= x\left[\left(1+\frac{1}{2}\tan^2\alpha\right)+32E^2x^2/3L^4+4Ex\tan\alpha/L^2\right]; \qquad \text{(15a)}
\end{aligned}
$$

And assuming that with two hangers per 6m pipe the points of attachments on the pipes are spaced by 3.015 m, so that their x coordinates are 3.015cos α, we get:

$$
l_{nc} = 3.015N\cos\alpha[1+\frac{1}{2}\tan^2\alpha+4\frac{E\tan\alpha}{L^2}\left(3.015N\cos\alpha\right)+\frac{32}{3}\frac{E^2}{L^4}\left(3.015N\cos\alpha\right)^2];
$$

Equation (15)

Note that in the last two equations the distances to the nth hanger on the left (n positive or up) and to the right (n negative or down) are not the same because of the sign of x and of n. Note also the sign of tanα in these equations. The distance ln_c is larger on the high side for a given absolute value of n. If the pipeline is bowed upwards by C_2, the horizontal distances corresponding to equal lengthwise spacing along the unbowed pipe are no longer equal, though the difference is minimal and can practically always be neglected.

<u>Anchoring the main cable.</u>

a) The first principle about anchoring the main cable into the ground is that <u>any bridge</u> no matter how small requires such an anchoring on both sides. Since in its absence a lateral force results on the top of the column, with a column base surrounded by frequently rain-saturated earth, tilting of the pilar towards the span will inevitably result in time. Even a simple corner fence post supporting barbed wire is always guyed for the same reason.

b) The second principle is that it is far better to fix the main cable and separately the anchoring cable to the top of the pillars than to let a single cable joining anchor to the main bridge span slide over the pillar top (short of a pulley). The reason is that in the first case, assuming that the pillar will tend to tilt infinitesimally in the direction of an initial side-force only a vertical force will result onto the pillars, while in the second case under the same assumption an unbalanced horizontal friction component will subsist even if on the anchor side the cable angle to the horizontal is the same as on the bridge span side, which is usually impractical. Thus if the two cables are tied separately the size of the pillar and of its base can be minimized.

c) The dead-weight of concrete and/or stone required to insure a proper anchoring is determined according to the following considerations:

There are two predictable force components to be countered: The vertical component of the anchoring cable tension and the horizontal component of the same force. The vertical component will tend to lift the anchor mass and the horizontal one to slide it. In order to counter the sliding force with (static) friction one has to provide a normal (vertical)downward force and it turns out that this force is usually quite a bit larger than that required to balance the vertical component of the cable tension. Thus the net normal force provided by the weight of the anchor is the sum of that required to balance the vertical component of cable tension and that required to provide a normal force which gives rise to a sufficient static friction to prevent horizontal sliding of the anchor. The additional resistance created by the material at the end of the anchor which would have to be displaced in case of horizontal anchor motion is usually treated as a (substantial) factor of safety. The friction factor used for the soil is conservative because it is poorly known.

For a two-dimensional case, (single anchor on either side in line with a single main cable) let θ_a be the angle of the anchor cable to the horizontal and T_a that cable tension. The vertical pull of this cable is $T_a \sin\theta_a$, the horizontal pull $T_a \cos\theta_a$. The weight of the anchor must satisfy the inequality:

$$W_a \geq T_a \left\{ \frac{\cos\theta_a}{\mu} + \sin\theta_a \right\};$$

where a value of the friction factor μ=0.35 seems conservative for most soils. If the anchor is made of concrete, the volume corresponding to W can be taken as

$$\text{Anchor volume, } (m^3) = W_a(\text{kgs})/2200$$

The value of T_a is obtained by equating horizontal forces on the pilar, i.e

$$T_a=T_0/\cos\theta_a=WL/(8E\cos\theta_a)$$

so that if the anchor cable angle does no exceed 25^0 (see below) the weight of the largest anchor does not exceed

$$0.41WL/E$$

Now, the vertical force downward on the pillars is the sum of the vertical components of T_c and of T_a :

$$F_v= WL/8E(\tan\theta_a+ 4E/L+/-\tan\alpha)$$

The value of θ_a is best kept reasonably low to minimize both the above expression for anchor weight and the vertical force on the pillars as well as to keep the diameter of the anchor cable conveniently the same as that of the main cable. In practice, a value equal to or less than 25^0 to the horizontal seems adequate.

<u>Double Anchors.</u> With a single cable there are no planned side forces. But these can occur as a result of strong wind or impact from large flotsam and the columns are not guyed against such forces. So it is generally a good idea except for short bridges to provide two anchors on each end, disposed symmetrically with respect to the vertical plane of symmetry of the bridge.

Let us call the direction cosines of the two anchoring cables assumed symmetrically disposed with respect to the vertical center-plane of the main cable, respectively (in the x,y, z directions) α_a, β_a, $\pm\gamma_a$.
let these be defined in terms of the the two angles θ_y, in the x-y plane and θ_z , in the x-z plane, (these would be the angles whose tangents would be E/L_{ax}, $\pm Z_0/L_{ax}$ if the anchoring points were on the same level as the base of the support). The relation between these angles and the direction cosines is

$$\alpha_a = \frac{1}{\sqrt{1+(\tan\theta_y)^2+(\tan\theta_z)^2}};\ \beta_a = \alpha_a \tan\theta_y;\ \gamma_a = \alpha_a \tan\theta_z$$

Now the force relations between the main cable and the two anchor cables are

$$T_0= 2T_a\alpha_a \quad \text{or } T_a=T_0/2\alpha_a$$
with
$$T_{ax}=T_0/2;\quad T_{ay}=T_a\beta_a;\quad T_{az}=T_a\gamma_a$$

The vertical force is

$$T_{ay}=T_a\beta_a$$

and the horizontal force is

$$T_a(\alpha_a{}^2+\gamma_\alpha{}^2)^{1/2}$$

The weight of each anchor has to be at least

$$W=\{\text{horizontal force }/\mu\text{ +vertical force}\}$$

$$=\frac{T_0}{2\alpha_a}\left\{\frac{\sqrt{\alpha_a^2+\gamma_a^2}}{\mu}+\beta_a\right\}$$

and since T_0= WL/8E, if we take μ= 0.35 as before and choose $\tan\theta_y=\tan\theta_z=0.5$, we get for the weight of each anchor:

$$W_a=.23WL/E$$

and for their volumes:

$$V_a=1.05\ E^{-4}WL/E$$

<u>Additional details:</u>

a) As mentioned in previous notes one should never immobilize the pipes at the two ends of the bridge in concrete blocks. The reason is that in that case any horizontal side force on the pipeline, such as a wind or a striking branch might cause, will result in an almost unbounded inward force on the pipe and columns resulting in the failure of one, the other or both. Instead one allows the pipe to slide over a block at one end at least and one provides through two right angle turns a sufficient leeway for an inelastic deformation of the pipe beyond the bridge. The elbows used for the two right angle turns should be made of iron rather than plastic.

b) If pipes are hauled up by passing a thin cable over the major cable with a bending angle $\Delta\theta$, the hauling force T_2 will be greater than the weight raised W_1 by the ratio

$$T_2/W_1= e^{\mu\Delta\theta}$$

where μ, the coefficient of friction varied from 0.140 to 0.190 in my experiments (1/8” cable over 1/2” cable) and $\Delta\theta$ is the cable turn angle over the other cable in radians, (approximately π in many cases).

c) We often don’t have accurate values for the weight of our steel pipe, (often the major weight in a suspension aqueduct). Jordan[1]’s values apply to pipes used in Nepal which have different
d) thicknesses than the ones we use. It is thus appropriate to weigh the pipes provided. Their maximum working pressure can then be calculated simply by reference to the data below, (appendix 2) on Schedule 40 or Schedule 80 steel pipes, keeping in mind that the working pressure is very nearly directly proportional to the weight.

Appendix 1

Typical working strength of wire ropes
(steel cables) and their weight

d ”	d mm	Tp,kg	Cable weight, kg/m
1/16”	1.59	39.6	**0.009**
1/8”	3.175	158	**0.037**
3/16”	4.762	355	**0.083**
1/4"	6.35	632	**0.148**
5/16”	7.94	1105	**0.231**
3/8”	9.53	1424	**0.333**
7/16”	11.1	1930	**0.452**
½”	12.7	2498	**0.592**
5/8”	15.9	3891	**0.928**
¾”	19.05	5551	**1.332**
7/8”	22.3	7523	**1.825**
1”	25.4	9754	**2.368**
1 1/8”	28.6	1227	**3.002**
1 ¼”	31.7	15073	**3.690**
1 3/8”	34.5	18134	**4.370**

Appendix 2

Weight, steel pipes,schedules 40&80

Diameter nom.	Weight kg/m	Externl diam ,m	Wall thickness,mm	Internal diam.mm	Type,Sch.	Weight of water kg/m	Pipe + water, kg/m	Max. pressure,m
½"	1.27	0.021	2.78	15.44	40	0.187	1.457	425
1/2"	1.82	0.021	3.734	13.532	80	0.144	1.964	586
3/4"	1.68	0.027	2.87	21.26	40	0.355	2.035	339
3/4"	2.2	0.027	3.912	19.176	80	0.289	2.489	468
1"	2.51	0.033	3.378	26.244	40	0.541	3.051	318
1"	3.235	0.033	4.547	23.906	80	0.449	3.684	439
1 1/4"	3.385	0.042	3.558	34.884	40	0.956	4.341	262
1 1/4"	4.464	0.042	4.851	32.298	80	0.819	5.283	367
1 1/2"	4.048	0.048	3.683	40.634	40	1.297	5.345	233
1 1/2"	5.409	0.048	5.08	37.84	80	1.125	6.534	326
2"	5.441	0.06	3.912	52.176	40	2.138	7.579	212
2"	7.48	0.06	5.537	48.926	80	1.880	9.360	299
2 1/2"	8.63	0.073	5.158	62.684	40	3.086	11.716	198
2.1/2"	11.411	0.073	7.01	58.98	80	2.732	14.143	279
3"	11.284	0.089	5.488	78.024	40	4.782	16.066	184
3"	15.592	0.089	7.61	73.78	80	4.276	19.868	260
4"	16.073	0.114	6.02	101.96	40	8.166	24.239	155
4"	22.318	0.114	8.58	96.84	80	7.366	29.684	220

Note: If you do not obtain the characteristics of the HG pipes from your supplier, weigh them. Their working pressure is very nearly proportional to their weight, while their water capacity is nearly the same for all ratings, given the nominal diameter.

The program ABRIDGE

This program (presently in French and in Spanish can can be downloaded from our website and allows a quasi automatic design of suspension pipe bridges. It is based on the above notes.

[1] A Handbook of Gravity –Flow Water Systems by Thomas D. Jordan, *Intermediate Technology Publications, 1984*